A Quantitative Theory of Heterogeneous Returns to Wealth*

Guido Menzio
NYU and NBER

Saverio Spinella
NYU

December 2025

Abstract

We develop a macroeconomic model in which households are subject to uninsurable shocks to their endowment of labor, as in Aiyagari (1994), the financial market where household lend and firms borrow capital is subject to search frictions, as in Burdett and Judd (1983), and households invest in their ability to search. The model generates dispersion in the returns offered by firms for equally risky assets, persistent heterogeneity in the returns earned by households holding equally risky portfolios, and a positive correlation between wealth and risk-adjusted returns. These objects are endogenous, and depend on the marginal product of capital, the inflation rate, and the distribution of financial knowledge among households. A calibrated version of the model is used to quantify the effect of monetary, technology and policy shocks on financial market outcomes and, in particular, on the relationship between wealth and returns.

JEL Codes: D83, D52, E21, E44

Keywords: Search frictions, Financial frictions, Incomplete markets, Wealth inequality, Inflation.

1 Introduction

Recent empirical evidence documents that different individuals earn very different rates of return on their wealth (see Fagereng et al. 2020). Part of the heterogeneity in rates of return may be due to differences in the riskiness of the assets held by different individuals.

*Guido Menzio: Department of Economics, New York University, 19 West 4th Street, New York, NY 10012 (email: gm1310@nyu.edu). Saverio Spinella: Department of Economics, New York University, 19 West 4th Street, New York, NY 10012 (email: ss16213@nyu.edu). For their comments, we are grateful to Andy Atkeson, Anmol Bandhari, Jess Benhabib, Alberto Bisin, Corina Boar, Nicoló Ceneri, Luigi Guiso, Bob Hall, Joachim Hubmer, Erik Hurst, Sasha Indarte, Greg Kaplan, Ricardo Lagos, Paolo Martellini, Virgiliu Midrigan, Jaromir Nosal, Serdar Ozkan, Marco Pagano, Guillaume Rocheteau, Stefanie Stancheva, Sharon Traiberman, Gustavo Ventura, Gianluca Violante, Susan Woodward, and audiences at NYU, EIEF, Boston College, Rochester, SED Copenhagen, and at the Tribute to Bob Hall conference at Stanford. We are especially grateful to Wei Cui and Jonathan Parker for discussing our paper.

Part of the heterogeneity may be due to ex-post differences in the return of assets that, ex-ante, have the same risk and expected return. Part of the heterogeneity may be due to better terms for larger asset purchases. Yet, Fagereng et al. (2020) show that, in a 12-year panel regression of individual returns which controls for individual fixed effects, portfolio composition, portfolio's beta, and wealth percentiles, there is a great deal of variation in individual fixed effects. An individual at the 10% percentile of the fixed-effect distribution earns a return that is 3 percentage point below average. An individual at the 90th percentile of the fixed-effect distribution earns a return that is 3.5 percentage points above average. The dispersion of fixed-effects is sizeable within different subgroups of individuals (e.g., those who do and those who do not own any private equity) and different asset classes (e.g, safe and risky financial assets, private equity, housing). The dispersion of individual fixed-effect is positively correlated with wealth.

In this paper, we propose a theory of persistent heterogeneity in rates of return. The findings in Fagereng at al. (2020) imply the coexistence of assets that deliver different expected returns for the same level of risk, which, in turn, suggests that the Law of One Price does not hold in the retail asset market. Several empirical studies corroborate this view. For instance, Hortaçsu and Syverson (2004) document dispersion in annual fees and, hence, net returns across ostensibly identical S&P500 index funds. Woodward and Hall (2012) document dispersion in mortgage brokerage fees. In our theory, the Law of One Price fails in the financial market because of information frictions—modelled as in the canonical theory of price dispersion by Butters (1977), Varian (1980), and Burdett and Judd (1983). Households are not informed about all the financial intermediaries, but only from a discrete subset of them. Since some households are only informed about one and others households are informed about multiple intermediaries, equilibrium features dispersion in offered returns.

The findings in Fagereng et al. (2020) also suggest that some individuals are better than others at navigating the retail asset market. Several empirical studies support this view. For instance, Lusardi and Mitchell (2023) document heterogeneity in financial literacy. Clark, Lusardi and Mitchell (2015) document a positive relationship between financial literacy and returns to wealth. Cota and Šterc (2025) document a negative relationship between financial literacy and mortgage rates. In our theory, some households earn systematically higher returns than others because they are better at gathering information in a financial market where the Law of One Price fails. The ability of gathering information is a type of human capital that is endogenously accumulated over time, just like the human capital in Ben Porath (1966). Individual returns are positively correlated with wealth because wealthier households choose to invest more in their financial human capital.

Our theory is embedded in an otherwise standard macroeconomic model. Specifically, we consider a macroeconomic model in which households face uninsurable labor income risk, as in Aiyagari (1994), Bewley (1993), and Huggett (1996), and in which the financial market where households lend their savings to firms (either directly or through financial intermediaries) is subject to information frictions, as in Butters (1977), Varian (1980), and Burdett and Judd (1983). Outside of the financial market, households can invest their savings by holding fiat money. Inside of the financial market, households lend their savings to firms who use them as capital for production. If the financial market were perfectly competitive, firms would offer a return equal to the marginal product of capital, and all households would earn the same return on their wealth. Because of information frictions, however, the financial market is not perfectly competitive and firms offer rates below the marginal product of capital. Moreover, firms choose to offer different interest rates. Firms that offer lower interest rates borrow mainly from households whose only alternative is keeping their savings in cash. Firms that offer higher interest rates borrow also from households whose best alternative is lending their savings to a worse firm. The heterogeneity in interest rates offered by firms translates into heterogeneity in returns earned by households. And households with more financial human capital, who are aware of more investment opportunities, earn systematically higher returns.

In the first part of the paper, we use the theory to show, through a series of partial equilibrium exercises, that the dispersion in the interest rates offered by firms, the dispersion in the rates of return earned by households, and the relationship between individual returns and wealth are not immutable objects, but endogenous outcomes. We consider a shock to inflation. Since inflation is the negative of the return that households earn on their savings outside of the capital market, an increase in inflation allows firms to lower the distribution of real interest rates offered to households. The decline in the interest rates offered by firms is different in different parts of the distribution. At the bottom of the distribution, where firms are more likely to compete against the household's outside option, the decline is larger. At the top of the distribution, where firms are more likely to compete against each other, the decline is smaller. As a result, the decline in the returns to wealth is different for different types of households. For households with more financial human capital, households who are better informed and, thus, more likely to lend to firms at the top of the distribution, the decline is smaller. For households with less financial human capital, households who are less informed and, thus, more likely to lend to firms at the bottom of the distribution, the decline is larger. If financial human capital is positively correlated with wealth, inflation steepens the relationship between wealth and individual returns.

We consider a positive shock to the marginal product of capital. An increase in the marginal product of capital induces firms to offer higher interest rates to households. The increase in the interest rates offered by firms is different in different parts of the distribution. At the bottom of the distribution, where firms are more likely to compete against the household's outside option, the increase is smaller. At the top of the distribution, where firms are more likely to compete against other firms, the increase is larger. As a result, the increase in the returns to wealth is different for different households. For households with more financial human capital, the increase is larger, as these households are more likely to lend to firms at the top of the distribution. For households with less financial human capital, the increase is smaller, as these households are more likely to lend to firms at the bottom of the distribution. If financial human capital is positively correlated with wealth, a positive shock to the marginal product of capital steepens the relationship between wealth and individual returns.

We consider a shock to the wealth distribution. The distribution of wealth across households with different financial human capital is a key determinant of competition in the financial market. If a larger fraction of the wealth is owned by households with low financial human capital, the market becomes less competitive, as these households are less informed. If a larger fraction of the wealth is owned by households with high financial human capital, the market becomes more competitive, as these households are better informed. In the limit where all the wealth is in the hands of perfectly informed households, the market is perfectly competitive. Any shock that reallocates wealth from low to high financial human capital households induces firms to offer higher interest rates and, for this reason, it allows all types of households to earn higher rates of return on their wealth.

In the second part of the paper, we calibrate our model to Norwegian data. First, we ask the model to reproduce the distribution of fixed-effects from the panel regression in Fagereng et al. (2020). This target is informative about the distribution of households over financial human capital and, in turn, about the parameters that describe the stochastic process for the accumulation of financial human capital. We target moments of labor earnings from Halvorsen et al. (2024). These targets are informative about the parameters that describe the stochastic process for labor earnings. The calibrated model implies that financial human capital is very persistent over time. The calibrated model generates a positive relationship between individual wealth and returns, which is broadly consistent with the findings in Fagereng et al. (2020). The calibrated model generates a wealth distribution that is very similar to the actual wealth distribution, except at the very top.

We use the calibrated model to quantify the general equilibrium effects of monetary and technology shocks. A permanent increase in inflation from 2 to 10% lowers the interest rate offered by firms at the 10th percentile of the distribution by 4.7 percentage points, and it increases the rate offered by firms at the 90th percentile by 0.4 percentage points. It lowers the return earned by households at the 10th percentile of the wealth distribution by 4.1 percentage points, and the return earned by households at the 90th percentile of the wealth distribution by 5 basis points. By steepening of the relationship between wealth and returns, the increase in inflation leads to a sizeable increase in the extent and persistence of wealth inequality. The welfare cost of the increase in inflation is equivalent to 4% of consumption, and similar for poor, middle-class, and rich households.

A temporary 3% increase in total factor productivity increases the interest rate offered by firms at the 10th percentile by 8 basis points, and the rate offered by firms at the

90th percentile by 33 basis points. It increases the return earned by households at the 10th percentile of the wealth distribution by 13 basis points, and the return earned by households at the 90th percentile of the wealth distribution by 30 basis points. Richer households end up accumulating more wealth because of the positive productivity shock than poorer households. The welfare of richer households increases more than the welfare of poorer households because of the shock.

Using the calibrated model, we show how a literacy program that subsidizes investment in financial human capital can make the financial market almost perfectly competitive: most firms offer an interest rate that is nearly equal to the marginal product of capital, most households earn approximately the same return on their wealth, and the relationship between wealth and returns is essentially flat. Wealth inequality declines. By shrinking the wedge between marginal product of capital and returns earned by households, the literacy program leads to a long-run increase in aggregate output of 4.5%, and to a long-run increase in aggregate consumption of 2.2%.

Related literature. In this paper, we propose a theory of persistent heterogeneity in returns to wealth building on the price dispersion theory of Butters (1977), Varian (1980), and Burdett and Judd (1983) and on the financial human capital theory proposed by, among others, Jappelli and Padula (2013) and Lusardi, Michaud and Mitchell (2017). Some papers posit the existence of an increasing function mapping wealth to returns (see, e.g., Benhabib, Bisin and Luo 2019) or financial literacy to returns (see, e.g., Lusardi, Michaud and Mitchell 2017). In our theory, the relationship between wealth and returns and financial human capital and returns is endogenous and responds to monetary shocks, technology shocks and policy. Some papers derive persistent heterogeneous returns by assuming that firms differ in their productivity and have limited access to capital (see, e.g., Cagetti and DiNardi 2006, Boar, Gorea and Midrigan 2022, Halvorsen et al. 2024, Benhabib, Cui and Miao 2024). These theories explain heterogeneity in returns for owners of private equity. While heterogeneity in returns for owners of private equity is larger than in the general population, Fagereng et al. (2020) show that such heterogeneity in returns is a ubiquitous phenomenon. Some papers generate heterogeneous returns by positing a fixed cost to access some financial markets (see, e.g., Chatterjee and Corbae 1992, Kaplan and Violante 2014). These limited-participation models differ from ours because markets are perfectly competitive even if access to them is costly. Closest to our theory is Hortaçsu and Syverson (2004) who use a version of Burdett and Judd (1983) to explain the dispersion of net returns offered by different index funds. McKay (2013) is in the same vein. In these papers, there is no accumulation of financial human capital. In Section 4.3, we also consider a version of the model in which firms can discriminate households based on their wealth. We show that, if wealth and search skills are positively correlated, richer households earn higher returns both because they are better at searching and because they are offered higher interest rates.

The paper contributes to the growing literature that applies the search-theoretic model of imperfect competition of Burdett and Judd (1983) to macroeconomic questions (Head et al. 2012, Kaplan and Menzio 2016, Burdett and Menzio 2018, Pytka 2018, Nord 2023, Hubmer and Nord 2024, Sangani 2023, Menzio 2023, Albrecht, Menzio and Vroman 2023, Menzio 2024). We characterize the response of the distribution of offered rates to shocks to the return that households can earn outside of the market, and to the value of capital to firms. We show that these shocks have a different effect at different quantiles of the distribution of offered rates and, for this reason, they have a heterogeneous effect on the returns earned by households with different search skills. These results are cast in the context of a financial market. Yet, they have immediate counterparts in labor and product markets and, hence, they can be applied more generally.

The paper contributes to the literature exploring the link between monetary policy and frictional financial markets. In Silveira and Wright (2016), money is the medium of exchange in a frictional market for venture capital. In Lagos and Zhang (2020), money is the medium of exchange in a frictional asset market. In Cui, He and Wright (2024), money is the medium of exchange in a frictional market for used physical capital. In these models, inflation affects real money balances and, in turn, it affects quantities and prices in financial markets where money is the medium of exchange. In our model, money affects outcomes in the financial market not because it is used as the medium of exchange, but because money is the households' investment option outside of the financial market. In this sense, our model is related to Lagos and Zhang (2022), where money acts as a discipline device on banks. Indeed, in our model as in Lagos and Zhang (2022), inflation affects allocations even when real balances are vanishing (see Menzio and Spinella 2025). In other papers, frictions in financial markets are modelled as fixed trading costs (see, e.g., Chatterjee and Corbae 1992, Kaplan and Violante 2014, or Kaplan, Moll and Violante 2018). In Chatterjee and Corbae (1992), which is closest to us, money is a store of value that some households choose in oder to avoid paying a fixed cost to access the financial market. Inflation increases the fraction of households that choose to pay the fixed cost and, for this reason, it lowers real rates in the financial market.

2 Environment and Equilibrium

In this section, we present our model, which is essentially a version of the incomplete-markets model of Aiyagari (1994), Bewley (1983) and Huggett (1996), in which the capital market where firms rent and households lend capital is decentralized and frictional as in Butters (1977), Varian (1980) and Burdett and Judd (1983). In the capital market, firms offer interest rates to households. Because of frictions, households cannot lend to any firm, but only to the discrete subset of firms that they contact. The number of firms that a household contacts depends on the household's financial human capital, which is accumulated over time through costly investments, as in Lusardi, Michaud, and Mitchell (2017). The household's investment option outside the capital market is money. In Section

2.1, we describe the environment. In Section 2.2, we formulate the problem of a firm. In Section 2.3, we formulate the problem of a household. In Section 2.4, we formulate the market clearing conditions, and define an equilibrium.

2.1 Environment

The economy is populated by households, firms, and a government. There is a double continuum[1] of infinitely-lived, ex-ante identical households with measure 1. Each household maximizes the expected sum of current and future periodical utilities $u(c)$ discounted at the factor $\beta \in (0,1)$, where $u(c)$ is a strictly increasing and strictly concave function of consumption $c \in R_+$. Ex-post households are heterogeneous with respect to their endowment of efficiency units of labor $z \in \mathcal{Z}$, wealth $a \in \mathcal{A}$, and financial human capital $\lambda \in \Lambda$, with $\mathcal{Z} = \mathbb{R}_+$, $\mathcal{A} = \mathbb{R}_+$, and $\Lambda = \{\lambda_1, \lambda_2, ..., \lambda_I\}$. Households are endowed with ownership of the firms.[2]

There is a continuum of short-lived, ex-ante identical firms with an endogenous measure $\theta \geq 0$. Each firm has to pay a fixed cost $\zeta > 0$ to become active, where ζ are units of the consumption good. Each active firm operates a constant returns to scale technology that turns capital $k \in \mathbb{R}_+$ and efficiency units of labor $\ell \in R_+$ into $y(k,\ell) + (1-\delta_k)k$ units of the consumption good, where $y(k,\ell)$ is output, and $(1-\delta_k)k$ is undepreciated capital, which can be turned back into the consumption good at the rate of 1 for 1. The production function $y(k,\ell)$ is strictly increasing and strictly concave in k and ℓ. The parameter $\delta_k \in (0,1)$ is the depreciation rate of capital.

Within a period, events unfold as follows. First, the labor market opens. The labor market is centralized and frictionless. Households supply efficiency units of labor, and active firms demand efficiency units of labor according to the amount of capital that they borrowed in the previous period. Both households and firms take as given the real wage w, where w is such that the aggregate supply and the aggregate demand of efficiency units of labor are equated.

Second, production takes place. An active firm that borrowed k units of capital in the previous period and hired ℓ efficiency units of labor in the current period produces and sells $y(k,\ell)+(1-\delta_k)k$ units of the consumption good, and bears the fixed cost ζ. The firm pays $w\ell$ units of output to its workers. The firm pays rk units of the consumption good to its lenders, where r is the gross real interest rate that the firm promised in the previous period. The firm rebates any profits or losses to its owners, and then permanently exits the economy.

Third, the government injects additional fiat money into the economy through a lump-sum transfer to the households, or it withdraws some fiat money from the economy through a lump-sum tax on the households. Let M denote the stock of fiat money at the beginning

[1] Formally, there is a measure $1/\theta$ of households per firm, where θ denotes the measure of firms.

[2] How the ownership of firms is distributed across households is immaterial, since, in equilibrium, firms earn zero profits.

of the period, and let γ denote the net growth rate of fiat money. For $\gamma > 0$, the government injects $M\gamma$ units of fiat money through a lump-sum transfer to the households. The real value of the transfer to each household is $M\gamma\phi$, where ϕ denotes the price of a unit of fiat money in terms of the consumption good. Similarly, for $\gamma < 0$, the government withdraws $M\gamma$ units of fiat money through a lump-sum tax on the households. The real value of the tax on each household is $-M\gamma\phi$. In either case, a household receives a net transfer $T = M\gamma\phi$ from the government, where T is negative or positive depending on the sign of γ. After receiving T, a household allocates its resources into consumption $c \in R_+$, expenditures in financial education $e \in R_+$, and savings $s \in R_+$.

Next, the capital market opens. The capital market is decentralized and frictional. Firms decide whether or not to become active. Each active firm posts a gross real interest rate r. Each household comes into contact with a number of firms that depends on its financial human capital λ. In particular, a household with financial human capital λ comes into contact with n firms, where $n \in \{0, 1, 2, ...\}$ is drawn from a Poisson distribution with coefficient λ.[3] The household observes the real interest rate offered by the n firms that it has contacted, and decides whether and where to invest its savings s. Firms collect the savings from the household and turn them into capital at the rate of 1 for 1.

If a household does not lend its savings to a firm, it can either store the consumption good or hold fiat money. If the household stores the consumption good, it enjoys the real interest rate $1 - \delta_c$, where $\delta_c \in [0, 1]$ is the depreciation rate of consumption when stored across periods. If the household holds fiat money, it enjoys a real rate equal to $\hat{\phi}/\phi$, where $\hat{\phi}$ is the price of a unit of fiat money in terms of the consumption good in the next period, and ϕ is the price of fiat money in terms of the consumption good in the current period. The real interest rate $\underline{r}$ enjoyed by a household that does not lend its savings to a firm is the maximum between the return on storage $1 - \delta_c$ and the return on money $\hat{\phi}/\phi$.

Lastly, the household's idiosyncratic shocks for next period are realized. The household's endowment of efficiency units of labor in the next period is $\hat{z}$ with probability $\omega_z(\hat{z}|z)$, where z denotes the household's endowment of efficiency units in the current period. The household's financial human capital in the next period is $\hat{\lambda}$ with probability $\omega_\lambda(\hat{\lambda}|\lambda, e)$, where λ is the household's financial human capital in the current period, and e is the household's investment in financial education in the current period.

The environment described above is a version of the incomplete-markets model of Aiyagari (1994), Bewley (1983), and Huggett (1996), in which the market where households lend and firms borrow capital is decentralized and frictional, rather than Walrasian. Specifically, we assume that households cannot lend to just any borrower in the market, but only to a discrete subset of them. The assumption is meant to capture the view that

[3] We assume that the measure of firms θ does not affect the number of firms that a household contacts in the capital market. Theoretically, it would be easy to relax this assumption. Empirically, though, it would be difficult to identify the elasticity of the household's contacts with respect to the measure of firms.

households have a limited ability to locate investment opportunities, and they have a limited capacity to understand the investments offered by different borrowers. As a result, households are only able to lend to a limited subset of borrowers. We assume that the size of the choice set of a household is a random variable n whose average, λ, depends on the household's financial human capital—a measure of the household's knowledge about the financial market. We assume that a household's investment option outside of the capital market is either money (in a monetary equilibrium) or storage of the consumption good (in a non-monetary equilibrium). The assumption is natural. In a monetary equilibrium, it is natural to think that the household is paid in money and, hence, it can always hold its savings in cash. In a non-monetary equilibrium, the household is paid in the consumption good and, hence, it can always save by storing the good.

The capital market in this paper is modelled after the product market in Butters (1977), Varian (1980), and Burdett and Judd (1983). For this reason, the equilibrium in the capital market in this paper will share some of the features of the equilibrium in the product market in those papers. The fact that some households have access to a single investment opportunity will imply that, in equilibrium, borrowers offer interest rates that are below the marginal product of capital. The fact that some households have access to multiple investment opportunities will imply that, in equilibrium, the distribution of interest rates offered by borrowers is non-degenerate. The gap between the interest rates offered by borrowers and the marginal product of capital will depend on the average size of the households' choice sets (which determines the extent of competition between firms) and on the rate that households can earn outside of the capital market (which determines the extent of competition between firms and the households' outside option). The average size of the households' choice sets will be endogenous and depend on the households' investments in financial education.

In order to focus on the role of imperfect competition in the capital market, we made several simplifying assumptions—some innocuous and some less. We abstracted from financial intermediation. In particular, we assumed that households directly lend capital to firms. Introducing intermediaries that collect savings from households and deliver them to firms would not change the equilibrium conditions, as long as the market where households and intermediaries trade is decentralized and frictional, and the market where intermediaries and firms trade is centralized and frictionless. We abstracted from heterogeneity in the return and risk of different investments. In particular, we assumed that all firms operate the same non-stochastic production function. Extending the model to allow for heterogeneity and randomness in the production function would be easy, but outside the scope of this paper. We assumed that firms are short-lived. We made the assumption so as to guarantee that the revenues and the expenses of the firm occur in the same period and, hence, they can be evaluated without specifying the firm's discount factor, which cannot be easily recovered from the owners' preferences in a model where households are heterogeneous. Lastly, we assumed that households decide how much to consume and how much to save before they search the capital market. Modifying the model to allow households to make their consumption/saving decisions after searching the capital market is doable, but would significantly complicate the problem of the firm.

2.2 Problem of the firm

We now want to derive the equilibrium conditions. In this subsection, we formulate the problem of the firm in the labor and capital markets. In subsection 2.3, we formulate the problem of the household. In subsection 2.4, we deal with the clearing conditions in the labor, capital, and money markets. In subsection 2.5, we describe some of the key properties of equilibrium. We restrict attention to steady-state equilibria and, for this reason, we omit the dependence of value and policy functions on the aggregate state of the economy.

Consider a firm that rented k units of capital at the gross interest rate r in the capital market. The firm's problem in the labor market is

$$\max_{\ell \geq 0} y(k, \ell) + (1 - \delta_k)k - w\ell - rk - \zeta. \tag{2.1}$$

The firm's revenues are given by the sum of the revenues from selling its output $y(k, \ell)$ and the revenues from selling its undepreciated capital $(1 - \delta_k)k$. The firm's costs are given by the wage bill $w\ell$, the capital bill rk, and the fixed cost ζ. The firm chooses how many efficiency units of labor ℓ to hire in order to maximize its profit, which is given by the difference between revenues and costs.

The optimality condition for the firm's problem in the labor market is

$$y_2(k, \ell) = w, \tag{2.2}$$

where $y_2(k, \ell)$ denotes the derivative of the production function $y(k, \ell)$ with respect to its second argument. Solving the optimality condition (2.2) with respect to ℓ yields

$$\ell = g(w)k, \tag{2.3}$$

where $g(\cdot)$ denotes the inverse of the function $y_2(1, \cdot)$. The above expressions are easy to understand. The firm finds it optimal to hire a quantity of labor ℓ such that the marginal product of labor, the left-hand side of (2.2), is equal to the wage, the right-hand side of (2.2). Since the firm's production function features constant returns to scale in labor and capital, the marginal product of labor is homogeneous of degree 0 in ℓ and k. That is, $y_2(k, \ell)$ is equal to $y_2(1, \ell/k)$. Therefore, the optimal quantity of labor hired by the firm is equal to $g(w)k$, and it is linear in the firm's capital.

Substituting (2.3) into the firm's profit function (2.1) yields

$$\begin{aligned} & y(k, g(w)k) + (1-\delta_k)k - wg(w)k - rk - \zeta \\ = \; & y\,(1, g(w))\,k + (1-\delta_k)k - y_2(1, g(w))g(w)k - rk - \zeta \\ = \; & y_1(1, g(w))k + (1-\delta_k)k - rk - \zeta \\ = \; & (r^* - r)\,k - \zeta. \end{aligned} \tag{2.4}$$

The second line in (2.3) makes use of the fact that $y(k, \ell)$ has constant returns to scale and, hence, it is homogeneous of degree 1 in ℓ and k, and the fact that $w = y_2(1, g(w))$. The third line makes use of the fact that $y(k, \ell)$ has constant returns to scale and, hence, $y(1, g(w)) = y_1(1, g(w)) + y_2(1, g(w))g(w)$. The last line is obtained by defining r^* as the marginal product of capital, i.e.

$$r^* = y_1(1, g(w)) + 1 - \delta_k. \tag{2.5}$$

Overall, the maximized profit for a firm that has rented k units of capital at the gross interest rate r is equal to k times the difference between the marginal product of capital r^* and r minus the fixed cost ζ.

Next, we turn to the firm's pricing problem in the capital market. To formulate the problem of the firm, we need to introduce some notation. Specifically, we let $F(r)$ denote the fraction of firms that offer an interest rate smaller or equal to r, we let $F(r-)$ denote the fraction of firms that offer an interest rate strictly smaller than r, and we let $\chi(r)$ denote the measure of firms that offer an interest rate equal to r. Moreover, we let h_i denote the measure of households with financial human capital λ_i, and we let $H_i(s)$ denote the fraction of these households with savings that are smaller or equal to s.

Consider a firm offering the interest rate $r \geq \underline{r}$. The profit of the firm is given by

$$\pi(r) = \sum_{i=1}^{I} \left[\sum_{n=0}^{\infty} \int \frac{h_{i,n}}{\theta} \mu_{i,n}(r)\,(r^* - r)\,s dH_i(s) \right] - \zeta, \tag{2.6}$$

where

$$h_{i,n} = h_i \frac{e^{-\lambda_i} \lambda_i^{n+1}}{(n+1)!}\,(n+1)\,, \tag{2.7}$$

and

$$\mu_{i,n}(r) = F(r-)^n + \sum_{j=1}^{n} \binom{n}{j} \frac{F(r-)^{n-j} \chi(r)^j}{j+1}. \tag{2.8}$$

Let us explain the expressions above. The firm meets a measure $h_{i,n}/\theta$ of households with financial human capital h_i that are in contact with n other borrowers. The measure $h_{i,n}/\theta$ is given by the measure of households with human capital λ_i per firm, h_i/θ, times the probability that one of these households is in contact with $n+1$ borrowers (including the firm), $\exp(-\lambda_i)\lambda_i^{n+1}/(n+1)!$, times the number of borrowers contacted by each one of these households, $n+1$. The probability $\mu_{i,n}(r)$ that one of these households lends its savings to the firm is given by the sum of the probability of two events. The first event is that all the other n contacts of the household offer an interest rate strictly smaller than r. The second event is that j of the other n contacts of the household offer an interest rate equal to r, the remaining $n-j$ contacts of the household offer an interest rate strictly smaller than r, and the household randomizes in favor of the firm. If the household lends to the firm, the firm enjoys a profit of $(r^*-r)s$, where s are the household's savings. For any $r<\underline{r}$, the firm does not borrow any capital and $\pi(r)=-\zeta$.

The interest rate distribution F is consistent with firm's profit maximization if and only if (2.6) is maximized at every r on the support of F. Using this condition for the optimality of F, it is easy to show that F does not have any mass points (see, e.g., Lemma 1 in Menzio 2024). If F had a mass point at some $r_0\in[\underline{r},r^*)$, the firm's profit would be strictly greater at $r_0+\epsilon$ than at r_0, for some $\epsilon>0$ small enough and, hence, r_0 would not be a profit-maximizing interest rate. Indeed, by offering $r_0+\epsilon$, the firm would trade with all of the $h_{i,n}/\theta$ households that are in contact with j additional borrowers offering r_0 and with $n-j$ additional borrowers offering less than r_0. By offering r_0, the firm trades only with a fraction $1/(j+1)$ of the $h_{i,n}/\theta$ households that are in contact with j additional borrowers offering r_0 and with $n-j$ borrowers offering less than r_0. Hence, the firm raises discretely more capital by offering $r_0+\epsilon$ and, in doing so, it loses an arbitrarily small ϵ profit per unit of capital raised. The distribution F cannot have a mass point at some $r_0\geq r^*$ because no firm finds it optimal to offer an interest rate greater or equal than the marginal product of capital r^*. Indeed, by offering r_0, the firm's profit is no-greater than $-\zeta$. By offering $\underline{r}$, the firm's profit is strictly greater than $-\zeta$ because the firm can raise capital from the $h_{i,0}/\theta$ households who are not in contact with any other borrower. Similarly, F cannot have a mass point at any $r_0<\underline{r}$ because no firm finds it optimal to offer an interest rate strictly lower than the household's outside option $\underline{r}$.

Since F does not have any mass points, we can rewrite (2.6) as

$$\begin{aligned}
\pi(r) &= \sum_{i=1}^{I}\left[\sum_{n=0}^{\infty}\int\frac{h_i}{\theta}\frac{e^{-\lambda_i}\lambda_i^{n+1}}{n!}F(r)^n\,(r^*-r)\,sdH_i(s)\right]-\zeta\\
&= \frac{1}{\theta}\sum_{i=1}^{I}\left[\left(e^{-\lambda_i(1-F(r))}\lambda_i\sum_{n=0}^{\infty}\frac{e^{-\lambda_i F(r)}\lambda_i^n F(r)^n}{n!}\right)\int h_i\,(r^*-r)\,sdH_i(s)\right]-\zeta \qquad (2.9)\\
&= \frac{1}{\theta}\sum_{i=1}^{I}\left[e^{-\lambda_i(1-F(r))}\lambda_i S_i\,(r^*-r)\right]-\zeta.
\end{aligned}$$

The first line in (2.9) is obtained by substituting $h_{i,n}$ and $\mu_{i,n}(r)$ and by noting that $\chi(r)=0$. The second line is obtained by collecting terms. The last line is obtained by denoting with S_i the total savings of households with financial human capital h_i, i.e. $S_i=h_i\int sdH_i(s)$, and by noting that the summation over n equals 1.

The interest rate distribution F is consistent with firm's profit maximization if and only if (2.9) is maximized at every r on the support of F. Using this condition for the optimality of F, it is easy to show that the support of F is an interval $[r_\ell, r_h]$, with $r_\ell = \underline{r}$ (see, e.g., Lemma 2 in Menzio 2024). If the support of F had a gap between r_0 and r_1, with $r_0 < r_1$, the profit of the firm would be strictly greater at r_0 than at r_1. Indeed, the firm would raise the same amount of capital by offering r_0 and r_1, since $F(r_0) = F(r_1)$. The firm, however, would enjoy a strictly higher profit per unit of capital raised by offering r_0 rather than r_1. Therefore, the support of F must be an interval $[r_\ell, r_h]$. A similar argument can be used to show that the lowest interest rate on the support of F must be $\underline{r}$.

Since $\underline{r}$ is on the support of the interest rate distribution F, the firm attains its maximized profit, π^*, by offering $\underline{r}$. Since $\underline{r}$ is the lowest interest rate on the support of F, $F(\underline{r}) = 0$. These observations imply that

$$\pi^* = \frac{1}{\theta} \sum\nolimits_{i=1}^{I} \left[\lambda_i e^{-\lambda_i} S_i(r^* - \underline{r})\right] - \zeta. \tag{2.10}$$

Since r is on the support of the interest rate distribution F, the firm attains its maximized profit, π^*, by offering any $r \in [r_\ell, r_h]$. This observation implies that

$$\pi^* = \frac{1}{\theta} \sum\nolimits_{i=1}^{I} \left[\lambda_i e^{-\lambda_i (1-F(r))} S_i(r^* - r)\right] - \zeta. \tag{2.11}$$

Combining (2.10) and (2.11) yields

$$\sum\nolimits_{i=1}^{I} \lambda_i e^{-\lambda_i} S_i(r^* - \underline{r}) = \sum\nolimits_{i=1}^{I} \lambda_i e^{-\lambda_i (1-F(r))} S_i(r^* - r). \tag{2.12}$$

The expression in (2.12) is an equal-profit condition that uniquely pins down the equilibrium interest rate distribution F.

Lastly, we turn to the entry problem of firms. The measure θ of firms who choose to become active is such that

$$\zeta = \frac{1}{\theta} \sum_{i=1}^{I} \left[\lambda_i e^{-\lambda_i} S_i(r^* - \underline{r})\right]. \tag{2.13}$$

The left-hand side of (2.13) is the cost to the firm of becoming active. The right-hand side is the benefit to the firm of becoming active. Condition (2.13) states that the measure θ of active firms is such that the cost to a firm of becoming active must be equal to the benefit. From (2.13), it follows that firms make zero profits and, hence, the value of the firms is zero.

2.3 Problem of the household

Consider a household with wealth a, efficiency units of labor z, and financial human capital λ. The household's maximized lifetime utility $V(a, z, \lambda)$ is such that

$$
\begin{aligned}
& V(a, z, \lambda) \\
= \quad & \max_{(c,e,s)\in\mathbb{R}^3_+} u(c) + \beta\mathbb{E}_{\hat{z}|z}\left\{\sum_{\hat{\lambda}=\lambda_1}^{\lambda_I} \omega_\lambda(\hat{\lambda}|\lambda, e)\left[\sum_{n=0}^{\infty}\frac{e^{-\lambda}\lambda^n}{n!}\int V(sr, \hat{z}, \hat{\lambda})dF_n(r)\right]\right\}, \qquad (2.14)\\
& \text{s.t. } c + e + s \leq a + wz + T.
\end{aligned}
$$

In the current period, the household earns wz from supplying its efficiency units of labor and receives a transfer T from the government. The household chooses how to allocate these resources and its wealth a into consumption c, investment in financial human capital e, and savings s. In the capital market, the household meets n firms with probability $\exp(-\lambda)\lambda^n/n!$. For $n = 1, 2, \ldots$, the household's interest rate r is a draw from the cumulative distribution $F_n(r)$, where $F_n(r) = F(r)^n$ is the distribution of the highest of n draws from the distribution $F(r)$ of interest rates offered by firms. For $n = 0$, the household's interest rate r is a draw from $F_0(r)$, where $F_0(r)$ denotes a distribution that is degenerate at $\underline{r}$. In the next period, the household's financial wealth is sr. The household's efficiency units of labor are $\hat{z}$ with probability $\omega_z(\hat{z}|z)$. The household's financial human capital is $\hat{\lambda}$ with probability $\omega_\lambda(\hat{\lambda}|\lambda, e)$.

2.4 Market clearing and definition of equilibrium

The clearing condition for the capital market is

$$\sum\nolimits_{i=1}^{I}(1 - e^{-\lambda_i})S_i = K. \qquad (2.15)$$

The left-hand side of (2.15) is the capital that is lent by households to firms, which is given by the savings S_i of households with financial human capital λ_i multiplied by the fraction $1 - \exp(-\lambda_i)$ of these households that meet at least one firm. The right-hand side of (2.15) denotes the amount of capital borrowed by firms from households.

The clearing condition for the labor market is

$$L = g(w)K. \qquad (2.16)$$

The left-hand side of (2.16) is the amount of efficiency units of labor supplied by households, which we denote as L. The right-hand side of (2.16) is the amount of efficiency units of labor hired by firms, which is equal to the amount of capital borrowed by firms multiplied by $g(w)$.

The balanced-budget condition for the government is

$$M\gamma\phi = Th. \qquad (2.17)$$

The first-term on the left-hand side of (2.17) is the real value of the new money issued by the government in a given period. The second term on the left-hand side of (2.17) is the real value of the transfers made by the government to households in a given period. The sum of the two terms must be equal to zero.

The clearing condition for the money market depends on whether we are in a monetary equilibrium—a stationary equilibrium in which money has value—or in a non-monetary equilibrium—a stationary equilibrium in which money has no value.

For a monetary equilibrium to exist, it has to be the case that households prefer holding money than storing goods or, equivalently, $\hat{\phi}/\phi \geq 1 - \delta_c$. In this case, the clearing condition for the money market is

$$\sum\nolimits_{i=1}^{I} S_i e^{-\lambda_i} = M(1+\gamma)\phi. \tag{2.18}$$

The left-hand side of (2.18) is the real money demand by households, which is given by the savings S_i of households with financial human capital λ_i multiplied by the fraction $\exp(-\lambda_i)$ of these households that do not meet any firms. The right-hand side of (2.18) is the real value of the money supplied by the government.

We can then use the clearing condition for the money market in the next period to recover the real return on money. The clearing condition for the money market in the next period is

$$\sum\nolimits_{i=1}^{I} S_i e^{-\lambda_i} = M(1+\gamma)^2\hat{\phi}. \tag{2.19}$$

The demand for money from the households in the next period is the same as in the current period. The supply of money from the government increases by the factor $1+\gamma$, and the value of a unit of money is $\hat{\phi}$ rather than ϕ.

Equating the right-hand sides of (2.18) and (2.19) yields

$$\frac{\hat{\phi}}{\phi} = \frac{1}{1+\gamma}. \tag{2.20}$$

The real return on money is the inverse of the gross growth rate of the quantity of money in the economy or, equivalently, the inverse of the gross growth rate of the price of consumption in units of money (the inverse of the inflation rate). The expression in (2.20) implies that a monetary equilibrium may exist only if $1/(1+\gamma) \geq 1 - \delta_c$.

We are now in the position to define a stationary monetary equilibrium.

Definition 1. *A Stationary Monetary Equilibrium is a tuple* $\{\theta, F, r^*, \underline{r}, K, w, \phi, c, e, s, T, \mathcal{H}, S\}$ *such that: (i) The measure* θ *of firms satisfies the free-entry condition (2.13); (ii) The distribution* F *of interest rates offered is consistent with firms' profit-maximization and, hence, given by (2.12), where* r^* *is given by (2.5),* $\underline{r}$ *is given by* $1/(1+\gamma)$*, and* $1/(1+\gamma) \geq 1 - \delta_c$*; (iii) The aggegate capital* K *and the wage* w *satisfy the market clearing conditions (2.15) and (2.16); (iv) The price of money* ϕ *satisfies the market clearing condition (2.18); (iv) The policy functions* $c(a,z,\lambda)$*,* $e(a,z,\lambda)$ *and* $s(a,z,\lambda)$ *solve the problem of the* *household (2.14); (vi) The government's transfer T satisfies the balanced-budget condition (2.17); (vii) The distribution $\mathcal{H}(a, z, \lambda)$ of households is stationary; (viii) The savings S are consistent with the distribution $\mathcal{H}(a, z, \lambda)$ and the policy function $s(a, z, \lambda)$.*

In a non-monetary equilibrium, money has no value. In a non-monetary equilibrium, households prefer storing goods than holding money. When households prefer storing goods than holding money, the clearing conditions for the money market imply $\phi = 0$ and $\hat{\phi} = 0$. Hence, a non-monetary equilibrium may exist for any γ. In what follows, we shall assume that the economy is in a monetary equilibrium whenever the condition $1/(1 + \gamma) \geq 1 - \delta_c$ holds.

A stationary non-monetary equilibrium is formally defined below.

Definition 2. *A Stationary Non-Monetary Equilibrium is a tuple $\{F, r^*, \underline{r}, K, w, \phi, c, e, s, \theta, T, \mathcal{H}, S\}$ such that: (i) The measure θ of firms satisfies the free-entry condition (2.13); (ii) The distribution F of interest rates offered is given by (2.12), where r^* is given by (2.5) and $\underline{r}$ is given by $1 - \delta_c$; (iii) The aggegate capital K and the wage w satisfy (2.15) and (2.16); (iv) The price of money ϕ is 0; (iv) The policy functions $c(a, z, \lambda)$, $e(a, z, \lambda)$ and $s(a, z, \lambda)$ solve (2.14); (vi) The government's transfer T satisfies (2.17); (vii) The distribution $\mathcal{H}(a, z, \lambda)$ of households is stationary; (viii) The savings S are consistent with the distribution $\mathcal{H}(a, z, \lambda)$ and the policy function $s(a, z, \lambda)$.*

3 Properties of equilibrium

In this section, we discuss some of the key properties of equilibrium. In Section 3.1, we characterize outcomes in the capital market and, in particular, the distribution of interest rates offered by different firms and the rates of return earned by different households. This characterization allows us to lay out our theory of persistent heterogeneity in returns to wealth. We then show how outcomes in the capital market are affected by changes in the marginal product of capital, changes in the rate of return that households earn outside of the capital market, and changes in the distribution of savings across households. These partial equilibrium exercises illustrate the mechanics of our imperfectly competitive capital market. In Section 3.2, we characterize the solution of the household's problem and, in particular, how the household's savings are affected by financial human capital, and how the household's investment in financial human capital is affected by wealth.

3.1 Equilibrium in the capital market

Let $r(x)$ denote the interest rate offered by a firm at the x-th quantile of F, i.e. let $r(x)$ be implicitly defined by $F(r(x)) = x$. From (2.12), it follows that $r(x)$ is given by

$$r(x) = r^* - \frac{\sum_{i=1}^{I} \lambda_i e^{-\lambda_i} S_i}{\sum_{i=1}^{I} \lambda_i e^{-\lambda_i (1-x)} S_i} (r^* - \underline{r}). \tag{3.1}$$

The expression in (3.1) shows that $r(x) < r^*$ for all $x \in [0,1]$. That is, the interest rates offered by firms are strictly lower than the marginal product of capital r^*. Since a firm meets some captive households, which are households that can only lend to one borrower, the firm can guarantee itself strictly positive profits after entry and, hence, it offers an interest rate that is strictly lower than r^*. The expression in (3.1) also shows that $r'(x) > 0$ for all $x \in [0,1]$. Hence, the distribution of interest rates offered by firms is non-degenerate. Since a firm meets some non-captive households, which are households that can lend to multiple borrowers, the firm would have an incentive to outbid the competition if all the other firms offered the same interest rate. Lastly, the expression in (3.1) shows that $r(x) \geq \underline{r}$ for all $x \in [0,1]$. That is, the interest rates offered by firms are greater than the rate of return $\underline{r}$ that households can earn outside of the capital market. Since all households can obtain the return $\underline{r}$, firms must offer an interest rate greater of equal to $\underline{r}$. The firm at the bottom of the distribution offers $\underline{r}$, since this firm only borrows from captive households and, hence, it only competes against the households' outside option.

Using (3.1), we can compute the average rate of return $\hat{r}(n)$ earned by a household that comes into contact with n firms in the capital market. For $n = 0$, $\hat{r}(n)$ is equal to $\underline{r}$. For $n = 1, 2, \ldots$, $\hat{r}(n)$ is given by

$$\begin{aligned} \hat{r}(n) &= \int_{r_\ell}^{r_h} r dF(r)^n \\ &= \int_0^1 r(x) n F(r(x))^{n-1} F'(r(x)) r'(x) dx \\ &= \int_0^1 r(x) dx^n. \end{aligned} \tag{3.2}$$

The first line in (3.2) makes use of the fact that the distribution of the maximum of the interest rate offered by n firms is $F(r)^n$. The second line is obtained by changing the variable of integration from r to x. The last line makes use of the fact that $F(r(x)) = x$ and, hence, $F'(r(x))r'(x) = 1$. Since $r(x)$ is a strictly increasing function of x, and the cumulative distribution function x^n is strictly increasing in n (in the sense of first-order stochastic dominance), the last line in (3.2) implies that $\hat{r}(n+1) > \hat{r}(n)$ for $n = 1, 2, \ldots$ Moreover, since $r(x) > \underline{r}$ for all $x > 0$, it follows that $\hat{r}(1) > \hat{r}(0)$.

We can also compute the average rate of return $\hat{r}(\lambda)$ earned by a household with financial human capital λ. Specifically, $\hat{r}(\lambda)$ is given by

$$\hat{r}(\lambda) = \sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} \hat{r}(n). \tag{3.3}$$

Since $\hat{r}(n)$ is strictly increasing in n and the Poisson distribution is strictly increasing in λ (in the sense of first-order stochastic dominance), (3.3) implies that $\hat{r}(\lambda)$ is strictly increasing in λ. In other words, households with more financial human capital earn, on average, a higher rate of return on their wealth than households with less financial human capital. Since an individual's financial human capital is a state variable, households with more financial human capital earn systematically higher rates of return than households with less financial human capital. If financial human capital is increasing in wealth, in the sense of first-order stochastic dominance, richer households earn systematically higher rates of return than poorer households. In a nutshell, this is our theory of persistent heterogeneity in individual rates of returns on wealth.

The following proposition summarizes our findings.

Proposition 1: (Interest rates). (i) *The distribution F of interest rates offered by firms is non-degenerate.* (ii) *The average rate of return on wealth $\hat{r}(n)$ earned by a household that contacts n firms in the capital market is such that $\underline{r} = \hat{r}(0) < \hat{r}(1) < \hat{r}(2) < .... < r^*$.* (iii) *The average rate of return on wealth $\hat{r}(\lambda)$ earned by a household that has financial human capital λ is such that $\underline{r} < \hat{r}(\lambda_1) < \hat{r}(\lambda_2) < ... < \hat{r}(\lambda_I) < r^*$.*

We now want to study the determinants of the interest rates offered by firms and of the rates of returns earned by households. First, we characterize the effect of an increase in the marginal product of capital r^*. Second, we characterize the effect of a decline in the rate of return $\underline{r}$ that households can earn outside of the capital market. Third, in a version of the model in which firms differ with respect to their total factor productivity, we consider the effect of stretching out the distribution of marginal products of capital.[4] Lastly, we characterize the effect of a change in the distribution of savings across households with different financial human capital. We carry out these exercises in partial equilibrium–in the sense that we treat r^*, $\underline{r}$, and the distribution of savings and financial human capital across households as parameters. In Section 5, we carry out general-equilibrium analogues to these partial equilibrium exercises.

Consider an increase in the marginal product of capital r^* due to, say, an increase in total factor productivity. The derivative with respect to r^* of the interest rate $r(x)$ offered by a firm at the x-th quantile of the F distribution is

$$\frac{dr(x)}{dr^*} = 1 - \frac{\sum_{i=1}^{I} \lambda_i e^{-\lambda_i} S_i}{\sum_{i=1}^{I} \lambda_i e^{-\lambda_i(1-x)} S_i}. \tag{3.4}$$

For $x = 0$, the derivative is equal to 0. For $x \in [0, 1]$, the derivative is strictly increasing. For $x = 1$, the derivative is strictly smaller than 1. An increase in r^* leads firms to increase the interest rate offered to households, but the pass-through is less than 1-for-1. The pass-through is zero for firms at the lowest quantile of the distribution, the pass-through is strictly increasing in the firms' quantile, and it is strictly smaller than 1 for firms at the highest quantile of the distribution. These findings are easy to understand. Firms at the lowest quantile of the distribution only compete against the household's outside option. Since the households' outside option is unchanged, these firms do not increase their interest rate. Firms at higher quantiles of the distribution are more likely to be competing against each another. Since the firm's profit margin increases with r^*, firms at higher quantiles of the distribution increase their interest rates more.

Since an increase in the marginal product of capital r^* leads to an increase in the

[4] We are grateful to Luigi Guiso for suggesting this exercise.

interest rates offered by firms, the average rate of return $\hat{r}(n)$ earned by a household that comes into contact with n firms increases as well. Moreover, since $dr(x)/dr^*$ is strictly increasing in x and the cumulative distribution function x^n is strictly increasing in n, (3.2) implies that $d\hat{r}(n+1)/dr^* > d\hat{r}(n)/dr^*$ for $n = 1, 2, \ldots$ Since $d\hat{r}(0)/dr^* = 0$ and $d\hat{r}(1)/dr^* > 0$, it follows that $d\hat{r}(1)/dr^* > d\hat{r}(0)/dr^*$. An increase in r^* benefits more households who contact more firms. This finding is intuitive. Households that contact more firms lend to firms at higher quantiles of the distribution. Firms at higher quantiles of the distribution increase their interest rate by more. Hence, households that contact more firms enjoy a larger increase in their average rate of return.

Similarly, an increase in the marginal product of capital r^* increases the average interest rate $\hat{r}(\lambda)$ earned by households with financial human capital λ. Moreover, since $d\hat{r}(n)/dr^*$ is strictly increasing in n and the distribution of contacts is strictly increasing in λ, it follows from (3.3) that $d\hat{r}(\lambda)/dr^*$ is strictly increasing in λ. An increase in r^* benefits more households that have more financial human capital. This finding is also intuitive. Households with more financial human capital contact more firms, lend to firms at higher quantiles of the distribution, and enjoy a larger increase in their rate of return.

The following proposition summarizes.

Proposition 2: (Interest rates and marginal product of capital). *The effect of a change in the marginal product of capital r^* is as follows*:

- **(i)** *The interest rate $r(x)$ offered by a firm at the x-th quantile of the F distribution is such that $dr(0)/dr^* = 0$, $dr(x)/dr^*$ is strictly increasing in x, and $dr(x)/dr^* < 1$.*
- **(ii)** *The average rate of return $\hat{r}(n)$ earned by a household that contacts n firms in the capital market is such that $0 = d\hat{r}(0)/dr^* < d\hat{r}(1)/dr^* < ....1$.*
- **(iii)** *The average rate of return $\hat{r}(\lambda)$ earned by a household that has financial human capital λ is such that $0 < d\hat{r}(\lambda_1)/dr^* < ... < d\hat{r}(\lambda_I)/dr^* < 1$.*

In a perfectly competitive capital market, an increase in r^* leads firms to increase 1-for-1 the interest rate offered to households, and it leads households to increase 1-for-1 the rate of return earned by households. Proposition 2 shows that this is not the case when the capital market is imperfectly competitive as it is in our model. An increase in r^* leads firms to increase the interest rate offered to households, but less than 1-for-1. The increase in interest rates is lowest at the bottom of the distribution, and highest at the top of the distribution. The increase in the rate of return earned by household with less financial human capital is smaller than the increase in the rate of return earned by household with more financial human capital. To the extent that households with more financial human capital are wealthier, an increase in r^* tends to increase inequality.

Now, consider a decline in the rate of return $\underline{r}$ that households can earn outside of the capital market. The derivative with respect to $\underline{r}$ of the interest rate $r(x)$ offered by a firm at the x-th quantile of the F distribution is

$$\frac{dr(x)}{d\underline{r}} = \frac{\sum_{i=1}^{I} \lambda_i e^{-\lambda_i} S_i}{\sum_{i=1}^{I} \lambda_i e^{-\lambda_i (1-x)} S_i}. \tag{3.5}$$

For $x = 0$, the derivative is equal to 1. For $x \in [0, 1]$, the derivative is strictly decreasing. For $x = 1$, the derivative is strictly greater than 0. A decline in $\underline{r}$ leads firms to lower the interest rate offered to households. The rate offered by firms at the bottom of the distribution declines 1-for-1 with the decline in the outside rate. The rate offered by firms at higher quantiles of the distribution declines by less. The rate offered by firms at the top of the distribution declines the least. These findings are easy to understand. Firms at the lowest quantile of the distribution only compete against the household's outside option. For this reason, these firms lower their interest rate by the same amount as the outside rate. Firms at higher quantiles of the distribution are more likely to be competing against each another. For this reason, these firms lower their interest rate by less than the outside rate.

Since a decline in the outside rate $\underline{r}$ leads to a fall in the interest rates $r(x)$ offered by firms, the average rate of return $\hat{r}(n)$ earned by a household with n contacts falls as well. Moreover, since $dr(x)/d\underline{r}$ is strictly decreasing in x and the cumulative distribution function x^n is strictly increasing in n, (3.2) implies that $d\hat{r}(n)/d\underline{r} > d\hat{r}(n+1)/d\underline{r}$ for $n = 1, 2, \ldots$ Since $d\hat{r}(0)/d\underline{r} = 1$ and $d\hat{r}(1)/d\underline{r} < 1$, it follows that $d\hat{r}(0)/d\underline{r} > \hat{r}(1)/d\underline{r}$. A decline in $\underline{r}$ hurts more households with fewer contacts in the capital market. This finding is intuitive. Households that do not contact any firms hold their savings outside of the capital market. Their rate of return falls 1-for-1 with $\underline{r}$. Households that contact some firms hold their savings in the capital market, where rates fall less than $\underline{r}$ does. Hence, the rate of return of these households falls less than 1-for-1 with $\underline{r}$. Households with more contacts in the capital market lend to firms at higher quantiles of the distribution, which are firms that lower their rates by less. Hence, the rate of return of these households falls by less. Since $d\hat{r}(n)/d\underline{r}$ is strictly decreasing in n and the distribution of contacts is strictly increasing in λ, (3.3) implies that $d\hat{r}(\lambda)/d\underline{r}$ is strictly decreasing in λ. A decline in $\underline{r}$ hurts more households that have less financial human capital.

The following proposition summarizes.

Proposition 3: (Interest rates and outside option) *The effect of a change in the interest rate $\underline{r}$ that households can obtain outside of the capital market is as follows:*

(i) *The interest rate $r(x)$ offered by a firm at the x-th quantile of the F distribution is such that $dr(0)/d\underline{r} = 1$, $dr(x)/d\underline{r}$ is strictly decreasing in x, and $dr(1)/d\underline{r} > 0$.*

(ii) *The average rate of return $\hat{r}(n)$ earned by a household that contacts n firms in the capital market is such that $1 = d\hat{r}(0)/d\underline{r} > d\hat{r}(1)/d\underline{r} > \ldots > 0$.*

(iii) *The average rate of return $\hat{r}(\lambda)$ earned by a household that has financial human capital λ is such that $1 > d\hat{r}(\lambda_1)/d\underline{r} > \ldots. > d\hat{r}(\lambda_I)/d\underline{r} > 0$.*

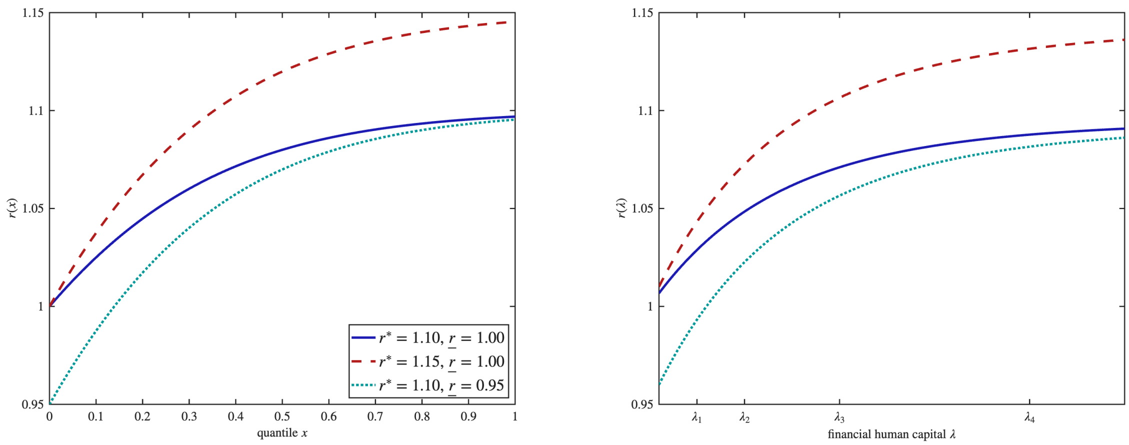
In a perfectly competitive capital market, a decline in the outside rate $\underline{r}$ does not lead to any change in the interest rates offered by firms, and, consequently, it does not lead to any change in the rates of return earned by households. Proposition 3 shows that this is not the case in our model. Consider a monetary equilibrium, where the real rate of return that households can earn outside of the capital market is the inverse of the inflation rate. An increase in inflation allows firms to offer lower real interest rates because they understand that holding cash is the only available alternative for some of their lenders.[5] The decline in the real interest rates offered by firms is different in different parts of the distribution. The decline is larger at the bottom of the distribution, where firms mostly borrow from households whose only alternative is cash, and it is smaller at the top of the distribution, where firms mostly borrow from households who have the option of lending to other firms. The decline in the real rate of returns earned by households is different for different types of households. Households with low financial human capital suffer a larger decline in their real rate of return. First, households with low λ are more likely to hold their savings in cash and, hence, to directly bear the decline in the real rate of return on money. Second, households with low λ contact fewer firms in the capital market and, hence, they lend to firms at lower quantiles of the distribution, which are firms that lower their real interest rates by more. As long as financial human capital and wealth are positively correlated, an increase in inflation tends to increase inequality.

Figure 1 illustrates the findings in Propositions 1, 2 and 3. Panel (a) plots the interest rate $r(x)$ offered by a firm at the x-th quantile of the F distribution for different values of the marginal product of capital r^* and for different values of the rate of return $\underline{r}$ that households can obtain outside of the capital market. Panel (b) plots the average rate of return $\hat{r}(\lambda)$ earned by households with financial human capital λ for different values of r^* and $\underline{r}$.

Both an increase in r^* and a decline in $\underline{r}$ magnify the difference between the returns earned by household with high and low financial human capital. There is a simple intuition behind this observation. Both an increase in r^* and a decline in $\underline{r}$ stretch out the distribution of interest rates offered by firms—in the sense that they increase the gap between the interest rate offered by a firm at the x_1-quantile and the rate offered by a firm at the x_0-quantile of the distribution for any $x_1 > x_0$. When the distribution of interest rates offered by firms is stretched out, the difference between the return earned by households who are better at searching the market (those with high financial human capital) and the return earned by households who are worse at searching the market (those with low financial human capital) increases.

There are other shocks that stretch out the distribution of interest rates offered by firms. To illustrate this point, let us consider a version of the model in which firms are

[5] Note that $\underline{r}$ affects the distribution of interest rates offered by firms because there are some households who contact only one firm in the capital market, not because there are some households who do not contact any firms. As shown in Menzio and Spinella (2025), this implies that monetary policy affects the capital market even when the demand for money is arbitrarily small.



(a) Distribution of offered rates $r(x)$

(b) Average earned rates $r(\lambda)$

Figure 1: Distribution of offered rates, and average earned rates by λ.

heterogeneous with respect to their marginal product of capital because, say, they differ in their total factor productivity. Let $\Phi(r^*)$ denote the distribution of firms across marginal product of capitals, and assume that $\Phi(r^*)$ is twice-continuously differentiable and its support is an interval $[r^*_\ell, r^*_h]$, with $\underline{r} < r^*_\ell < r^*_h$. Let $r^*(x)$ denote the marginal product for a firm at the x-th quantile of the distribution Φ.

Following the same steps as in Menzio (2024, Lemma 3), it is easy to show that the interest rate offered by a firm is a strictly increasing function of its marginal product of capital. Hence, $F(r(x)) = \Phi(r^*(x)) = x$ and, in turn, $F'(r(x))r'(x) = 1$. The interest rate $r(x)$ offered by a firm at the x-th quantile of the Φ distribution satisfies the first-order condition

$$\begin{aligned} & \sum\nolimits_{i=1}^{I} \lambda_i^2 e^{-\lambda_i(1-F(r(x)))} S_i F'(r(x)) \left(r^*(x) - r(x)\right) \\ = & \sum\nolimits_{i=1}^{I} \lambda_i e^{-\lambda_i(1-F(r(x)))} S_i. \end{aligned} \tag{3.6}$$

Using the fact that $F(r(x)) = x$ and $F'(r(x))r'(x) = 1$, we can rewrite (3.6) as

$$r'(x) = \left(\frac{\sum_{i=1}^{I} \lambda_i^2 e^{-\lambda_i(1-x)} S_i}{\sum_{i=1}^{I} \lambda_i e^{-\lambda_i(1-x)} S_i} \right) \left(r^*(x) - r(x)\right). \tag{3.7}$$

The expression in (3.7) is a differential equation for $r(x)$. The boundary condition associated with the differential equation is $r(0) = \underline{r}$, since the firm with the lowest marginal product of capital must offer the lowest interest rate and the lowest interest rate must be equal to the outside rate $\underline{r}$.

We want to understand the effect on the equilibrium of the capital market of stretching out the distribution of marginal products of capital across firms. Specifically, we want to compare the equilibrium outcomes in the capital market with $\Phi_1(r^*)$ and $\Phi_0(r^*)$ such that the associated quantile functions $r_1^*(x)$ and $r_0^*(x)$ have the properties $r_1^*(x) > r_0^*(x)$ for all $x \in [0,1]$, and $r_1^{*\prime}(x) \geq r_0^{*\prime}(x)$ for all $x \in [0,1]$.

First, we show that firms offer higher rates of return under the distribution $\Phi_1(r^*)$ than under the distribution $\Phi_0(r^*)$. That is, we show that $r_1(x) > r_0(x)$ for all $x \in (0,1]$. To this aim, consider any x_0 such that $r_1(x_0) = r_0(x_0)$. Since $r_1^*(x_0) > r_0^*(x_0)$, (3.7) implies that $r_1^{*\prime}(x_0) > r_0^{*\prime}(x_0)$, i.e. if the functions $r_1(x)$ and $r_0(x)$ ever cross, $r_1(x)$ crosses $r_0(x)$ from below. Hence, $r_1(x)$ and $r_0(x)$ can only cross at x_0, and $r_1(x) > r_0(x)$ for all $x > x_0$. Since $r_1(0) = \underline{r}$ and $r_0(0) = \underline{r}$, it follows that $r_1(x) > r_0(x)$ for all $x \in (0,1]$.

Second, we show that the derivative of the interest rate offered by a firm with respect to the firm's quantile x is higher under the distribution $\Phi_1(r^*)$ than under the distribution $\Phi_0(r^*)$. That is, we show that $r_1'(x) > r_0'(x)$. To this aim, let us define

$$v_0(x) = \left[\sum\nolimits_{i=1}^{I} \lambda_i e^{-\lambda_i(1-x)} S_i\right] \left(r_0^*(x) - r_0(x)\right). \tag{3.8}$$

Using the first-order condition (3.7), we find that the derivative of $v_0(x)$ with respect to x is

$$v_0'(x) = \left[\sum\nolimits_{i=1}^{I} \lambda_i e^{-\lambda_i(1-x)} S_i\right] r_0^{*\prime}(x). \tag{3.9}$$

Analogously, we can define $v_1(x)$ and compute $v_1'(x)$. Since $r_1^{*\prime}(x) \geq r_0^{*\prime}(x)$, it follows that $v_1'(x) \geq v_0'(x)$. Since $v_1(0) > v_0(0)$ and $v_1'(x) \geq v_0'(x)$, it follows that $v_1(x) > v_0(x)$ or, equivalently,

$$\begin{aligned} & v_1(x) - v_0(x) \\ = \; & \left[\sum\nolimits_{i=1}^{I} \lambda_i e^{-\lambda_i(1-x)} S_i\right] \left[\left(r_1^*(x) - r_1(x)\right) - \left(r_0^*(x) - r_0(x)\right)\right] > 0. \end{aligned} \tag{3.10}$$

The inequality above implies $r_1^*(x) - r_1(x) > r_0^*(x) - r_0(x)$. It then follows from (3.7) that $r_1'(x) > r_0'(x)$.

Since $r_1'(x) > r_0'(x)$ and $r_1(0) = r_0(0) = \underline{r}$, firms at higher quantiles of the distribution increase their interest rates by more than firms at lower quantiles of the distribution. Following the same steps as in Proposition 2, we can then show that households with more contacts enjoy a larger increase in their rate of return and, consequently, households with more financial human capital enjoy a larger increase in their rate of return.

Proposition 4: (Interest rates with heterogeneous marginal products of capital) *Let $\Phi_1(r^*)$ and $\Phi_0(r^*)$ be two distribution of marginal productivities of capital such that $r_1^*(x) > r_0^*(x)$ and $r_1^{*\prime}(x) \geq r_0^{*\prime}(x)$ for all $x \in [0,1]$. The equilibrium of the capital market has the following properties:*

(i) *For $k = 0,1$, let $r_k(x)$ denote the interest rate offered by a firm at the x-th quantile of the F distribution given $\Phi_k(r^*)$. Then $r_1(x) > r_0(x)$ for all $x \in (0,1]$, and $r_1'(x) > r_0'(x)$ for all $x \in [0,1]$.*

(ii) *For $k = 0,1$, let $\hat{r}_k(n)$ denote the average rate of return on wealth earned by a household that contacts n firms given $\Phi_k(r^*)$. Then $0 = \hat{r}_1(0) - \hat{r}_0(0) < \hat{r}_1(1) -$*

$\hat{r}_0(1) < ...$

(iii) *For* $k = 0, 1,,$ *Let* $\hat{r}_k(\lambda)$ *denote the average rate of return on wealth earned by a household that has financial human capital* λ *given* $\Phi_k(r^*)$. *Then* $0 < \hat{r}_1(\lambda_1) - \hat{r}_0(\lambda_1) < \hat{r}_1(\lambda_2) - \hat{r}_0(\lambda_2) < ....$

Proposition 4 considers an environment in which firms are heterogeneous with respect to their marginal product of capital. The proposition then shows that households with more financial human capital have more to gain in times when distribution of firms' returns is stretched out—in the sense that the marginal product of capital increases more at firms at the top of the distribution than at firms at the bottom of the distribution. In contrast, in a perfectly competitive market, only the firm with the highest marginal product of capital succeeds in borrowing from households and all households benefit equally from an increase in that firm's productivity.

In our model, the extent of competition in the capital market is endogenous, and, crucially, it depends on the distribution of savings acoss households with different financial human capital. Moving savings from households with relatively low financial human capital, i.e. households with a relatively low λ, to households with relatively high financial human capital, i.e. households with a relatively high λ, leads to an increase in the interest rates offered by firms. The finding is intuitive. If a larger fraction of savings is in the hands of households with high financial human capital, the capital market becomes more competitive, and the distribution of interest rates offered by firms increases (in the sense of first-order stochastic dominance). The finding is a version of previous results in Kaplan and Menzio (2016) and Nord (2023), showing that the competitiveness of the market depends on the (appropriately weighted) fraction of agents with low and high searching ability.

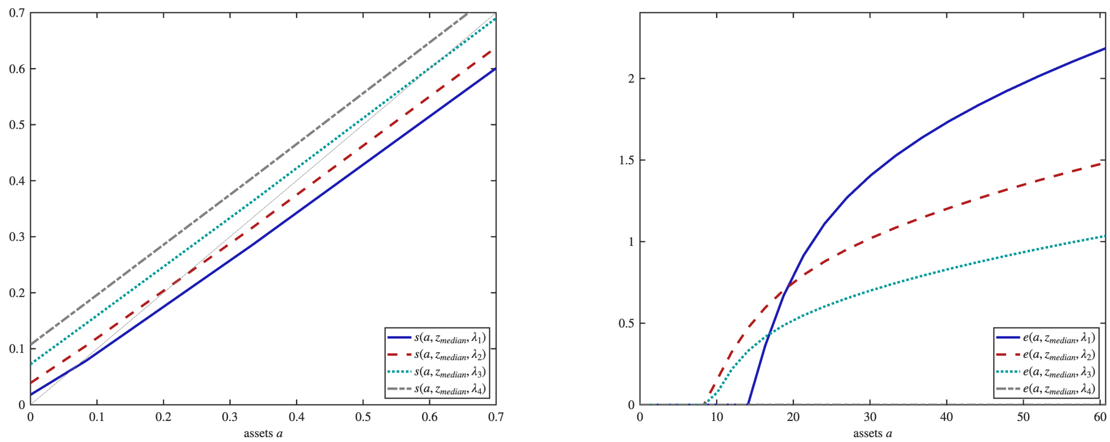
The proposition below formalizes the argument above. The proof is trivial.

Proposition 5. (Competition in the capital market). *Consider an increase* $dS_i > 0$ *in the savings of households with financial human capital* λ_i *that is compensated by a decline* $dS_j = -(\lambda_i \exp(-\lambda_i)/\lambda_j e \exp(-\lambda_j))dS_i$ *in the savings of households with financial human capital* λ_j, *where* $\lambda_j < \lambda_i$. *The interest rate offered by a firm at the* x*-th quantile of the* F *distribution is such that* $dr(x)/dS_i > 0$ *for all* $x \in (0, 1]$.

3.2 Household's behavior

Let us examine the saving behavior of households. The optimality condition for savings $s > 0$ is given by

$$\begin{aligned} & u'(a + wz + T - e - s) \\ \geq \quad & \beta \mathbb{E}_{\hat{z}|z} \left\{ \sum_{\hat{\lambda}=\lambda_1}^{\lambda_I} \omega_\lambda(\hat{\lambda}|\lambda, e) \left[\sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} \int r \frac{\partial V(sr, \hat{z}, \hat{\lambda})}{\partial a} dF_n(r) \right] \right\}, \end{aligned} \tag{3.11}$$



(a) Savings

(b) Investment in financial hc

Figure 2: Household's policy functions

and $s \geq 0$, where the two inequalities hold with complementary slackness. The left-hand side of (3.11) is the marginal cost of increasing s, which is given by the marginal utility of consumption in the current period. The right-hand side of (3.11) is the marginal benefit of increasing s, which is given by the expected marginal value of r additional units of wealth in the next period. Condition (3.11) states that the marginal cost of increasing savings must be equal to the marginal benefit, as long as $s > 0$. If $s = 0$, the marginal cost of increasing savings must be greater or equal to the marginal benefit.

The optimal choice of s depends on the household's current wealth a and on the household's current efficiency units of labor z, as it would in a standard incomplete-markets models in the style of Ayiagari (1994). In contrast to standard incomplete-markets models, the optimal choice of s also depends on the household's financial human capital λ. Figure 2(a) plots the household's optimal savings as a function of its current wealth a, for different levels of financial human capital. As expected, the household's optimal savings are strictly increasing in the household's wealth a, since higher a lowers the marginal cost of savings s. The household's optimal savings are increasing in the household's financial human capital λ, since financial human capital allows the household to contact more firms and obtain a higher interest rate.

Next, let us examine the household's investment in financial human capital. The optimality condition for e is given by

$$\begin{aligned} & u'(a + wz + T - e - s) \\ \geq \quad & \beta \mathbb{E}_{\hat{z}|z} \left\{ \sum_{\hat{\lambda}=\lambda_1}^{\lambda_I} \frac{\partial \omega_\lambda(\hat{\lambda}|\lambda, e)}{\partial e} \left[\sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!} \int V(sr, \hat{z}, \hat{\lambda}) dF_n(r) \right] \right\}, \end{aligned} \tag{3.12}$$

and $e \geq 0$, where the two inequalities hold with complementary slackness. The left-hand side of (3.12) is the marginal cost of increasing e, which is given by the marginal utility of consumption in the current period. The right-hand side of (3.12) is the marginal benefit of increasing e, which is given by the marginal change in the probability of having financial human capital $\hat{\lambda}$ in the next period multiplied by the expected continuation value conditional on $\hat{\lambda}$. The household's continuation value is increasing in $\hat{\lambda}$ because a household with more financial human capital contacts from firms and obtains higher interest rates. Condition (3.12) states that the marginal cost of investing more in financial human capital must be equal to the marginal benefit, as long as $e > 0$. If $e = 0$, the marginal cost of investing more in financial human capital must be greater or equal to the marginal benefit.

The optimal choice of e depends on the household's current wealth a and on the household's current efficiency units of labor z. Figure 2(b) plots the household's optimal investment in financial human capital as a function of its current wealth a. The household's optimal investment e is increasing in the household's wealth a, since higher a lowers the marginal cost of e and increases the marginal benefit of e. Figure 2(b) plots the household's optimal investment in financial human capital for different levels of financial human capital. The relationship between e and λ depends on the details of the stochastic process relating e and λ to $\hat{\lambda}$. In the case illustrated by Figure 2(b), households with more financial human capital choose lower e for sufficiently high levels of wealth.

The characterization of the household's behavior suggests that households with more financial human capital will tend to accumulate more wealth. Similarly, households with more wealth will tend to accumulate more financial human capital. Taken together, these two observations suggest that financial wealth and financial human capital will be positively (albeit, imperfectly) correlated.

4 Calibration

In this section we calibrate the model. In Section 4.1, we discuss the calibration strategy. To calibrate the parameters that describe the capital market, we target moments that summarize the extent to which different households earn systematically different rates of return on their wealth after controlling for differences in portfolio composition; moments about the fraction of wealth held in cash by different households; and moments about the persistence of wealth inequality. To calibrate the parameters that describe the labor market, we follow the incomplete-markets literature and target moments about individual labor earnings. In Section 4.2, we discuss some properties of the calibrated model. The model reproduces well the extent of persistent heterogeneity in rates of return, which is driven by heterogeneity in financial human capital. The model correctly predicts a positive relationship between rates of return and wealth, which is driven by the positive correlation between wealth and financial human capital. The model generates a realistic wealth distribution, except at the very top. In Section 4.3, we calibrate a version of the model in which firms can discriminate households based on wealth. In this version of the model, wealthier households earn higher rates both because they are better at searching the capital market and because they are offered higher interest rates.

4.1 Calibration targets

First, let us review the parameters of the model. The preferences of the household are described by the utility function $u(c)$, and by the discount factor β. We specialize the utility function to have the CRRA form $u(c) = c^{1-\nu}/(1-\nu)$, where ν is the coefficient of relative risk aversion. The technology of the firm is described by the production function $y(k, \ell)$, and by the depreciation rate of capital δ_k. We specialize the production function to have the Cobb-Douglas form $y(k, \ell) = Ak^{\alpha}\ell^{1-\alpha}$, where α is the elasticity of output with respect to capital, $1-\alpha$ is the elasticity of output with respect to labor, and A is total factor productivity. We normalize total factor productivity A to 1. The monetary policy of the government is described by the growth rate γ of the aggregate stock of money.

The stochastic process $\omega_z(\hat{z}|z)$ for the evolution of the household's endowment of efficiency units of labor is specialized to have the log AR(1) form $\log \hat{z} = \rho \log z + \epsilon$, where $\hat{z}$ denotes the household's efficiency units in the next period, z denotes the household's efficiency units in the current period, ρ is an autocorrelation parameter, and ϵ is a random variable that is normally distributed with mean μ_ϵ and standard deviation σ_ϵ. The realization of ϵ is independent across households and across time. We choose μ_ϵ so that the total measure L of efficiency units of labor supplied by households is 1.

The stochastic process for the evolution of the household's financial human capital is chosen to be a process of climbing up and down a ladder. The rungs of the ladder are equally spaced on a logarithmic scale. Specifically, the rungs of the ladder are given by $\Lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$, with $\lambda_1 = 0.5$, $\lambda_2 = 1$, $\lambda_3 = 2$, and $\lambda_4 = 4$. The stochastic process for the evolution of the household's financial human capital is $\omega_\lambda(\lambda_{\hat{\imath}}|\lambda_i, e)$, where $\lambda_{\hat{\imath}}$ denotes the household's financial human capital in the next period, λ_i denotes the household's financial human capital in the current period, and e denotes the household's investment in financial human capital in the current period. For $i = 2$, 3 and 4, we set $\omega_\lambda(\lambda_{i-1}|\lambda_i, e) = \delta_\lambda$, where δ_λ is a parameter that captures the probability of climbing down a rung of the ladder. For $i = 1$, 2 and 3, we set $\omega_\lambda(\lambda_{i+1}|\lambda_i, e) = \min\{1 - \exp(-\eta_i e), 1 - \delta_\lambda\}$, where η_i is a parameter than controls the effect of investment on the probability of climbing a rung of the ladder. For $i = 1$, 2, 3 and 4, we set $\omega_\lambda(\lambda_i|\lambda_i, e)$ to be $1 - \omega_\lambda(\lambda_{i+1}|\lambda_i, e) - \omega_\lambda(\lambda_{i-1}|\lambda_i, e)$. That is, a household's financial human capital goes up by a rung, goes down by a rung, or remains unchanged.

Next, let us turn to the calibration of the parameters of the model. We calibrate the central parameters of the model to Norwegian data over the period 2004-2015. We assume that the economy is in a stationary monetary equilibrium. We assume that the length of a period is 1 year. We set the coefficient ν of relative risk aversion in the household's preferences function to be 1.5, a common choice in the incomplete-markets literature. We set the elasticity $1-\alpha$ of output with respect to labor in the firm's production function to be 0.64 to guarantee that the share of GDP accruing to labor is 64%, a typical measure of the labor share. We set the depreciation rate of capital δ_k to be 8%, a typical measure of capital depreciation. We set the growth rate of the aggregate stock of money to be 2%. This implies that the inflation rate is 2%, which is approximately the average inflation rate in Norway over the period of interest.

We calibrate the parameters of the stochastic process for the household's efficiency units of labor to capture the properties of household's labor earnings. Specifically, we simulate the detailed labor earnings process estimated by Halvorsen et al. (2024) using Norwegian data. Then, we use the simulated data to estimate the autocorrelation ρ and the standard deviation σ_ϵ of our AR(1) process.

The remaining parameters are the coefficients η_1, η_2, and η_3 in the probability of climbing a rung of the financial human capital ladder with respect to the investment in financial human capital, the probability δ_λ of descending a rung of the human capital ladder, and the factor β at which households discount future utility. These parameters determine the outcomes in the financial market.

We jointly calibrate these parameters to match several targets. First, we target moments of the distribution of returns to net wealth documented by Fagereng et al. (2020) using Norwegian data. Fagereng et al. (2020) regress yearly individual returns on net wealth on individual fixed-effects, portfolio composition, portfolio's beta, and wealth percentile. Since the regression controls for portfolio composition and portfolio's beta, the fixed-effect describes the additional return that a particular individual earns on its net wealth for the same level of risk. Since the regression contains 11 years of observations per individual, the fixed-effect describes the additional return that a particular individual earns systematically relative to others. Fagereng et al. (2020) find that there is a great deal of heterogeneity in individual fixed-effects. An individual at the 10th percentile of the fixed-effect distribution earns a rate of return that is 3.03 percentage points lower than average. An individual at the 25th percentile of the fixed-effect earns a rate of return that is 1.58 percentage points lower than average. An individual at the 75th percentile of the fixed-effect distribution earns a rate of return that is 1.86 percentage points higher than average. An individual at the 90th percentile of the fixed-effect distribution earns a rate of return that is 3.5 percentage points higher than average. We use the 10th, 25th, 75th and 90th percentiles of the fixed-effect distribution as calibration targets.[6] These targets are informative about the distribution of households across rungs of the financial human capital ladder and, hence, about the ηs and δ_λ. We target the average rate of return across households, which Fagereng et al. (2020) find to be 1.5%, and the wealth-weighted average return across households, which they find to be 3.6%. These targets are

[6] We construct the individual fixed-effects as in Fagereng el al. (2020). Specifically, we simulate the model and build an 11-year panel of individual returns to wealth. We then regress the returns to wealth on individual fixed-effects and wealth percentiles. We do not regress the returns to wealth on portfolio composition and portfolio's beta, because all assets are equally risky in the model.

informative about the marginal product of capital and, hence, about β.

In order to further discipline the parameters of the financial human capital process, we target a moment related to the persistence of wealth. Halvorsen et al. (2024) use the same data as Fagereng et al. (2020) to compute the rate at which individuals transition from one percentile of the wealth distribution to another. We summarize the persistence of wealth by computing the associated Shorrocks index.

Lastly, we target moments of the distribution of money holdings documented by Fagereng et al. (2020). Individuals between the 20th and the 50th percentile of the wealth distribution hold 28% of their wealth in money (defined as cash or bank deposits). Individuals between the 50th and the 90th percentile of the wealth distribution hold 9% of their net wealth in money. Since the probability that a household holds cash depend on its financial human capital, these targets are informative about the joint distribution of wealth and financial human capital and, in turn, about the parameters of the financial human capital process. Including these targets is also important to make sure that the model captures the incidence of money holdings for poorer and richer households.

4.2 Properties of the calibrated model

Table 1 reports the targets of the calibration and their model-generated counterparts. As one can see, the model matches relatively well the calibration targets, even though there are fewer internally calibrated parameters (5) than targets (9). Table 2 reports the value of the calibrated parameters of the model. Table 2 shows that the accumulation process of financial human capital is very persistent. The calibrated probability that a household descends a rung of the human capital ladder is 0.72% per year. The calibrated probability that a household climbs a rung of the human capital ladder given an investment equal to 1% of average labor earnings is 0.55% if the household is at the first rung of the ladder, 0.80% if the household is at the second rung of the ladder, 0.85% if the household is at the third rung of the ladder. The discount factor is 0.955.

Figure 3(a) plots the c.d.f. of interest rates offered by firms in the capital market. The rate of return that households can earn outside of the capital market is -2%, the negative of the inflation rate. The marginal product of capital that firms earn on each unit of capital lent to them is 4.33%. The distribution of rates offered by firms lies between these two extremes. A firm at the 10th percentile of the distribution offers an interest rate of -0.42%. A firm at the 50th percentile offers an interest rate of 3.06%. A firm at the 90th percentile offers an interest rate of 4.04%. The dispersion of rates offered by firms to households for investing in an equally risky activity is large. It is interesting to compare the dispersion of rates in the model and the dispersion of fees charged by S&P 500 index funds (an ostensibly homogeneous financial product). Using US data from 1995 to 2000, Hortaçsu and Syverson (2004) document that an index fund at the 10th percentile of the fee distribution charges an annual fee of approximately 20 basis points, while an index

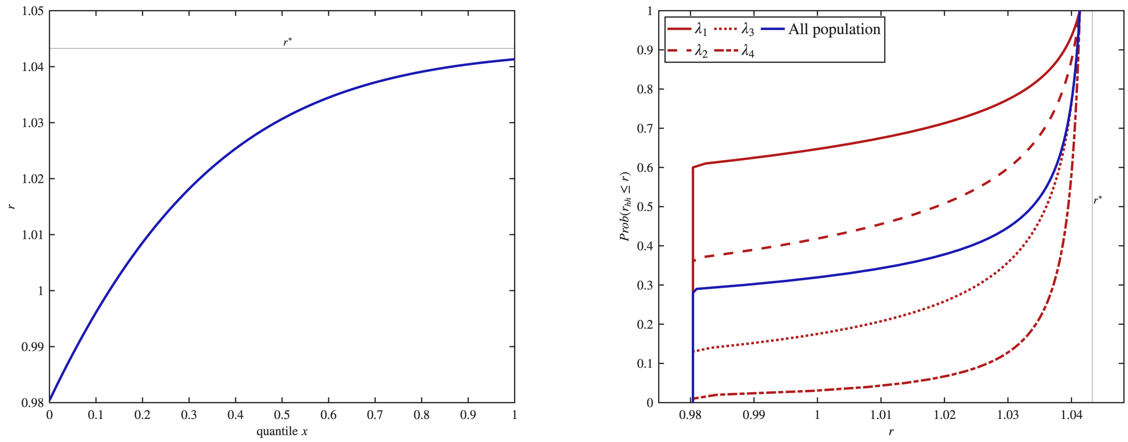
Table 1: Targets and fit

	Data	Baseline	Discrimination
Individual FEs Distribution			
10th percentile	-3.030	-2.651	-2.696
25th percentile	-1.580	-1.814	-1.695
75th percentile	1.860	1.887	1.693
90th percentile	3.500	2.209	2.053
Cash / Total Assets across Wealth Distribution			
20-50%	0.276	0.391	0.381
50-90%	0.090	0.097	0.090
$E_i(r)$	0.015	0.018	0.013
$E_s(r)$	0.036	0.032	0.034
Shorrocks index	0.588	0.380	0.370

fund at the 90th percentile of the distribution charges an annual fee of approximately 200 basis point. The 90-10 gap in offered returns is about 1.8 percentage points. In our model, the 90-10 gap is about 4 percentage points. The discrepancy between the predictions of our model and the findings in Hortaçsu and Syverson (2004) may be due to the origin of the data (Norway and US) or the type of financial product (a representative asset and an index fund).

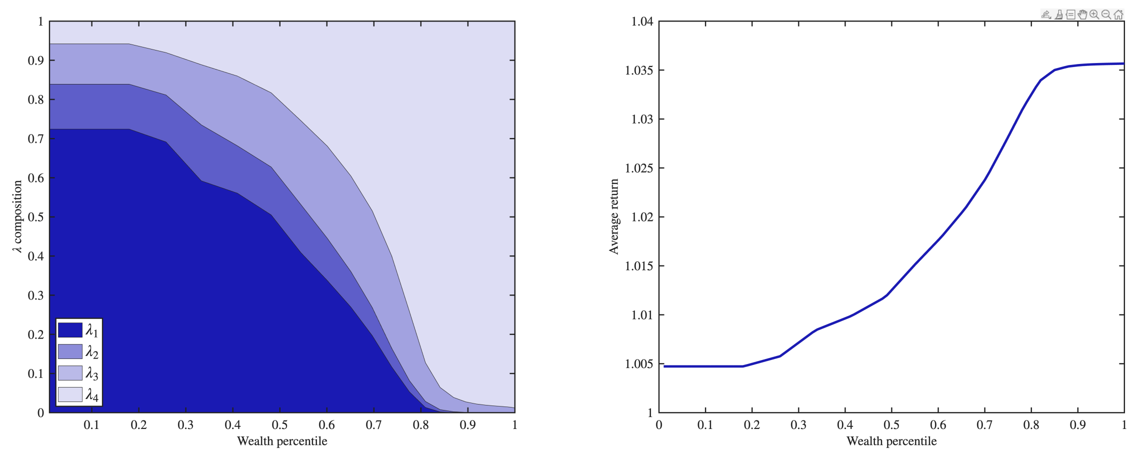
Figure 3(b) plots the c.d.f. of returns earned by households inside and outside of the capital market. About 29% of households keeps its savings outside of the capital market and earns the rate of return on money of -2%. Households at the 50th percentile of the distribution earn a rate of return of 3.33%. Households at the 90th percentile of the distribution earn a rate of return of 4.09%. Figure 3(b) also plots the c.d.f. of returns earned by households with different levels of financial human capital. The distribution of returns is increasing, in the sense of first-order stochastic dominance, in the financial human capital of households. The fraction of households that keep their savings in cash is 60% among those with the lowest financial human capital, and 2% among those with the highest financial human capital. At the 50th percentile of the respective distributions, a household with the lowest financial human capital earns -2%, and a household with the highest financial human capital earns 3.94%. At the 90th percentile of their respective distributions, a household with the lowest financial human capital earns 3.85%, and a household with the highest financial human capital earns 4.11%.

Figure 4(a) plots the distribution of financial human capital across households at different percentiles of the wealth distribution. It is evident that wealthier households



(a) Offered rates (b) Earned rates

Figure 3: Distribution of offered and earned rates



(a) Financial hc by wealth (b) Wealth and returns

Figure 4: Financial human capital and returns by wealth

Table 2: Calibrated Parameters

	Baseline	Discrimination	
Assigned			
A	1	1	TFP
δ_K	0.08	0.08	Physical capital depreciation rate
α	0.36	0.36	Output elasticity to physical capital
γ	0.02	0.02	Growth rate of money
ν	1.5	1.5	CRRA
Externally calibrated			
ρ_z	0.908	0.908	AR(1) coefficent $\log(z)$
σ_z	0.215	0.215	St.vdev. $\log(z)$ shocks
Internally calibrated			
η_1	0.468	0.516	Scale param $\lambda_1 \to \lambda_2$
η_2	0.726	0.795	Scale param $\lambda_2 \to \lambda_3$
η_3	0.731	0.795	Scale param $\lambda_3 \to \lambda_4$
δ_λ	0.007	0.007	FHC depreciation rate
β	0.955	0.954	Discount factor

tend to have more financial human capital. For example, at the 10th percentile of the wealth distribution, the fractions of households at the four rungs of the financial human capital ladder are, respectively, 72.4%, 11.5%, 10.3% and 5.8%. At the 90th percentile of the wealth distribution, the fractions of households at the four rungs of the ladder are, respectively, 0%, 0.1%, 2.7% and 97.2%. The positive relationship between wealth and financial human capital implies the positive relationship between wealth percentile and return on wealth illustrated in Figure 4(b). At the 10th percentile of the wealth distribution, households earn on average a return of 0.47% on their wealth. At the 90th percentile of the wealth distribution, households earn on average a return of 3.55% on their wealth. The 90-10 gap in returns is about 3 percentage points, and it is almost entirely accounted for by the difference in the average individual fixed-effects at different percentiles of the wealth distribution. For comparison, Fagereng et al. (2020) find that individual fixed-effects contribute about 2.5 percentage points to the 90-10 gap in returns.

The positive relationship between wealth and returns contributes to wealth inequality. Households below the 50th percentile of the wealth distribution own about 2.0% of the aggregate wealth. Households between the 50th and the 90th percentile own about 44.1% of the wealth. Households above the 90th percentile own the remaining 53.9% of the wealth. In Norway, households below the 50th percentile own 2.3% of the wealth. Households between the 50th and the 90th percentile own 46.5% of the wealth. Households above the 90th percentile own 51.2% of the wealth. At this level of aggregation, the model does a good job at matching the empirical concentration of wealth. The model, however, fails to capture the fraction of wealth in the hands of the very rich. In the model, the top 1% owns 11.3% of the wealth. In the Norwegian data, the top 1% owns 24.2% of the wealth. The failure of the model at matching wealth concentration at the top of the distribution should not be surprising. The model abstracts from non-homothetic preferences for bequest and private equity, two ingredients that have been shown to be critical to understand the right tail of the wealth distribution.

4.3 Wealth discrimination

Before turning to the applications of the calibrated model, we want to examine an alternative specification of the model. In the baseline model, we assume that firms cannot discriminate households based on their wealth—in the sense that firms offer the same interest rate to all households irrespective of their wealth s. In the baseline version of the model, wealthier households earn higher rates of return only because wealth and financial human capital are positively correlated and, hence, wealthier households contact more firms and lend to firms at higher quantiles. In this subsection, we develop and calibrate a version of the model in which firms can discriminate households based on their wealth—in the sense that they can condition the offered rate on the household's wealth s. In this version of the model, wealthier households earn higher rates of return not only because they tend to have more financial human capital and, hence, they lend to firms at higher quantiles, but also because firms at the same quantile offer them a higher interest rate.

When firms can discriminate households based on wealth, the distribution $F(r|s)$ of interest rates offered to households with wealth s is described by a quantile function $r(x|s)$ such that

$$r(x|s) = r^* - \frac{\sum_{i=1}^{I} h_i(s)\lambda_i e^{-\lambda_i}}{\sum_{i=1}^{I} h_i(s)\lambda_i e^{-\lambda_i(1-x)}}(r^* - \underline{r}). \tag{4.1}$$

where $h_i(s)$ denotes the fraction of households with wealth s that have financial human capital λ_i. The expression in (4.1) is derived from a version of the equal-profit condition (2.12) for households with wealth s. Note that, if the distribution of households across rungs of the financial human capital ladder is increasing in s (in the sense of first-order stochastic dominance), the quantile function $r(x|s)$ is increasing in s. This result is intuitive. If wealthier households have more financial human capital, firms have to compete more aggressively for their savings and, hence, they offer them higher interest rates.

The average rate of return earned by a household with n contacts depends on its wealth and it is given by

$$\hat{r}(n|s) = \int_0^1 r(x|s)dx^n. \tag{4.2}$$

The average rate of return earned by a household with financial human capital λ depends on its wealth and it is given by

$$\hat{r}(\lambda|s) = \sum_{n=0}^{\infty} \frac{e^{-\lambda}\lambda^n}{n!}\hat{r}(n|s). \tag{4.3}$$

If the distribution of households across rungs of the financial human capital ladder is increasing in s, $r(x|s)$ is increasing in s. Then, (4.2) implies that a wealthier household with n contacts earns, on average, a higher rate of return. Similarly, (4.3) implies that a wealthier household with financial human capital λ earns, on average, a higher rate of return.

The average rate of return for a household with wealth s is given by

$$r(s) = \sum_{i=1}^{I} h_i(s)\hat{r}(\lambda_i|s). \tag{4.4}$$

Assuming that the distribution of households across financial human capital ladder is increasing in s, $\hat{r}(\lambda|s)$ is increasing in s. Since $\hat{r}(\lambda|s)$ is increasing in s and λ, and $h_i(s)$ is increasing in s, (4.4) implies that households with more wealth earn, on average, a higher rate of return.

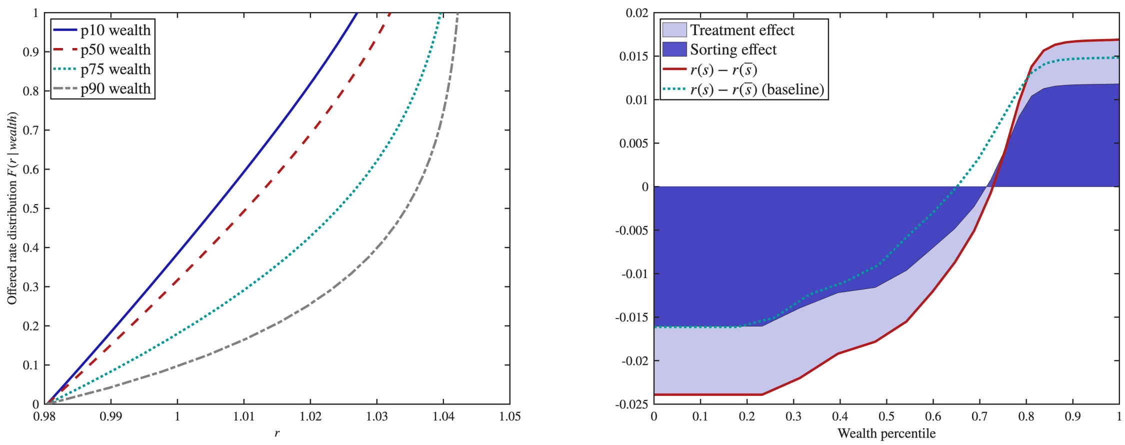
The average rate of return for a household with wealth s can be written as

$$\begin{aligned} \hat{r}(s) \quad &= \sum_{i=1}^{I} h_i(s)\hat{r}(\lambda_i|\overline{s}) \\ &+ \sum_{i=1}^{I} \overline{h}_i \left(\hat{r}(\lambda_i|s) - \hat{r}(\lambda_i|\overline{s})\right) + \sum_{i=1}^{I} (h_i(s) - \overline{h}_i)(\hat{r}(\lambda_i|s) - \hat{r}(\lambda_i|\overline{s})), \end{aligned} \tag{4.5}$$

where $\overline{h}_i$ denotes the fraction of households with financial human capital λ_i in the whole population, and $\overline{s}$ denotes the average wealth of households in the whole population. Assume that the distribution of households across the financial human capital ladder is increasing in s. Then, the first term on the right-hand side of (4.5) is increasing in s because $h_i(s)$ is increasing in s and $\hat{r}(\lambda|\overline{s})$ is increasing in λ. This term captures the indirect effect of wealth on returns: wealthier households have more financial human capital, they contact more firms and, hence, they manage to lend to firms that offer higher interest rates. We refer to this term as the *sorting effect* of wealth on returns. The second term on the right-hand side of (4.5) is increasing in s because $\hat{r}(\lambda|s)$ is increasing in s. This term captures the direct effect of wealth on returns: wealthier households are offered higher interest rates by firms. We refer to this term as the *treatment effect* of wealth on returns. The last term on the right-hand side of (4.5) is an interaction term between the two effects. In the baseline model, wealthier households earn higher returns only because of the sorting effect. In the model with discrimination, wealthier households earn higher also because of the treatment effect.[7] The interaction term may be increasing or decreasing in s, but the sum of the treatment effect and the interaction effect is always increasing in s.

We calibrate the wealth-discrimination model by matching the same empirical targets

[7] Ultimately, both the sorting and the treatment effect emerge because wealthier households have more financial human capital. As wealthier households have more financial human capital, they contact more firms and they lend to better firms (sorting). As wealthier households have more financial human capital, firms find it optimal to offer them higher rates (treatment).



(a) Distribution of offered rates by wealth

(b) Wealth and returns

Figure 5: Wealth discrimination

that we used to calibrate the baseline model.[8] The third column in Table 1 reports the model-generated analogues of the empirical targets. The second column in Table 2 reports the calibrated parameter values. Figure 5(a) plots the distribution of interest rates offered by firms to households with different levels of wealth. Since in equilibrium financial human capital is increasing in wealth, the distribution of interest rates offered by firms is increasing in the household's wealth. Figure 5(b) plots the relationship between households' wealth and average rate of return, and decomposes it into the sorting effect and the treatment effect (inclusive of the interaction term). Both effects contribute to the positive relationship between wealth and returns. The rate of return earned by a household at the 90th percentile of the wealth distribution is 4.06 percentage points higher than the rate of return earned by a household at the 10th percentile. Of those 4.06 extra percentage points, 1.29 are due to the treatment effect, and 2.77 are due to the sorting effect.

The relationship between wealth and rates of return is steeper in the version of the model with wealth discrimination than in the baseline model (see Figure 5b). For this reason, there is more wealth inequality in the version of the model with discrimination. The poorest 50% of households own 1% rather than 2% of the wealth. The poorest 90% of households own 42.2% rather than 46.1% of the wealth. The poorest 99% of the households own 87.7% rather than 88.7% of the wealth.

[8] The wealth-discrimination version of the model may admit multiple equilibria, since, as in a signaling model, the household's choice of savings affects the firm's beliefs about the household's financial human capital. We restrict attention to equilibria in which financial human capital is increasing in savings, in the sense of first-order stochastic dominance. Given this restriction, we find a unique equilibrium.

5 Applications

In this section, we use the calibrated baseline model to carry out three quantitative exercises.[9] In the first exercise, we examine the equilibrium and welfare effects of an increase in the inflation rate, brought about by an increase in the growth rate of money supply. This is a classic exercise in monetary theory. We find that inflation steepens the relationship between wealth and rates of return and, in doing so, it exacerbates wealth inequality. We find that inflation has large welfare costs, because it widens the wedge between marginal product of capital and households' returns. In the second exercise, we examine the equilibrium and welfare effects of a transitory shock to total factor productivity. This is a classic exercise in business cycle theory. We find that a positive productivity shock steepens the relationship between wealth and rates of return, and, in doing so, it ends up benefiting richer households more. In the third exercise, we examine the effect of a financial literacy program that subsidizes the households' investment in financial human capital. We find that the literacy program can make competition in the capital market essentially perfect, flatten the relationship between wealth and rates of return, significantly reduce wealth inequality, and increase both long-run output and consumption. These exercises provide a general equilibrium counterpart to Propositions 2, 3 and 5, which were derived in partial equilibrium.

5.1 Monetary policy

In this subsection, we use the calibrated model to measure the equilibrium and welfare effects of an increase in the growth rate of money supply. We consider a permanent and unanticipated increase in the growth rate of money supply from 2 to 10% per year. We measure the effect of the increase in the growth rate of money supply on equilibrium outcomes such as the distribution of interest rates offered by firms, the distribution of rates of returns earned by households, the distribution of financial human capital, the distribution of wealth, the aggregate capital and the real value of the stock of money. We also measure the effect of the increase in the growth rate of money supply on aggregate welfare, and on the welfare of different households.

In the long-run, the increase in the growth rate of money supply from 2 to 10% per year leads to an increase in the inflation rate from 2 to 10%. In the short-run, the increase in the growth rate of money supply generates even higher inflation rates, as the real value of the stock of money needs to transition towards its lower steady-state value. Higher inflation lowers the rate of return on money. A lower real rate of return on money allows firms to lower the interest rates that they offer to households—as firms understand that money is the only alternative investment for some of their lenders. The decline in the interest rates is more pronounced at the bottom of the distribution F—where firms are

[9] We decided to carry out these exercises using the baseline version of the model rather than the wealth-discrimination version of the model in order to keep the analysis simpler.

more likely to borrow from households whose only alternative is holding money—and less pronounced at the top of the distribution F—where firms are more likely to borrow from households who are in contact with multiple firms (see Proposition 3).

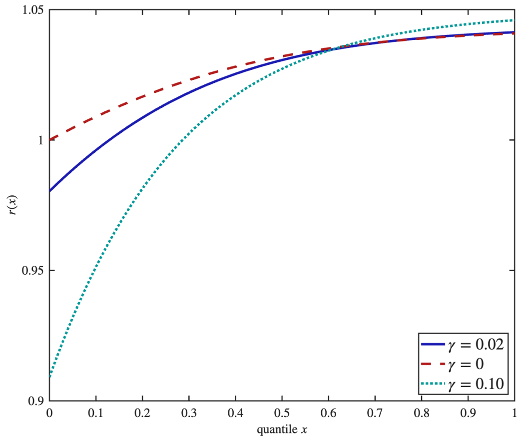
Figure 6(a) shows the long-run effect of the increase in the growth rate of money supply on the distribution of interest rates offered by firms (for comparison, we include the long-run distribution of interest rates when $\gamma = 0$). The median interest rate falls from 3.07% to 2.73%. The interest rate at the 10th percentile of the offer distribution falls from -0.49% to -4.88%. The interest rate at the 90th percentile of the offer distribution increases from 4.01% to 4.44%. The increase in the rate of return at the top of the distribution is caused by a long-run decline in the aggregate stock of capital, which leads to an increase in the marginal product of capital (from 4.33% to 4.90%).

The decline in the rates of return inside and outside the capital market has heterogeneous effects on the rates of return earned by different types of households. Households with less financial human capital are less likely to trade in the capital market and, hence, more likely to hold money. Conditional on trading in the capital market, households with less financial human capital are more likely to lend to firms at the bottom of the distribution. Since the decline in the rate of return of money is larger than the decline in the interest rates offered by firms in the capital market and since the decline in the interest rates offered by firms is larger at the bottom than at the top of the distribution, households with less financial human capital experience a larger decline in the rate of return on their wealth than households with more financial human capital (see, again, Proposition 3).

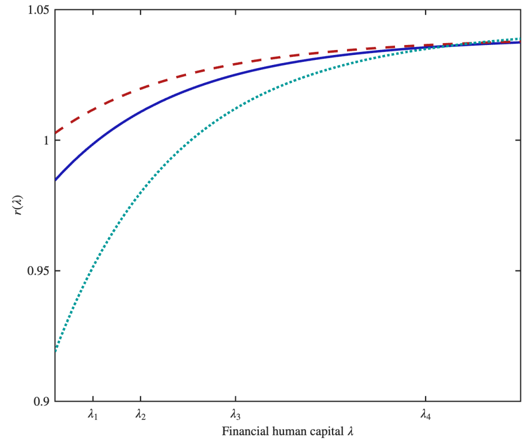
Figure 6(b) shows the long-run effect of the increase in the growth rate of money supply on the average rates of return earned by different types of households (for comparison, we include the rates of return for $\gamma = 0$). For households on the bottom rung of the financial human capital ladder, the average rate of return falls from -0.15% to -4.88%. For households on the second rung of the financial human capital ladder, the average rate of return falls from 1.08% to -2.01%. For households on the third rung of the ladder, the average rate of return falls from 2.51% to 1.12%. For households on the top rung of the ladder, the average rate of return slightly declines from 3.55% to 3.49%.

Since the gap between the average rate of return earned by a household with more financial human capital and the average rate of return earned by a household with less financial human capital grows, households have a stronger incentive to invest in financial knowledge. The increase in financial knowledge leads to a decline in the fraction of households that invest their wealth in money (because they fail to trade in the capital market), and to an increase in the fraction of households that invest their wealth in capital (because they manage to trade in the capital market). As a result, the fraction of aggregate wealth that households hold in cash declines, and the fraction of aggregate wealth that households invest in capital increases.

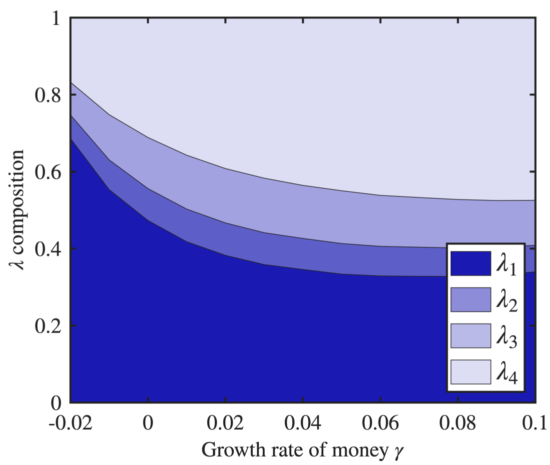
Figure 6(c) shows the long-run effects of the growth rate of money on the distribution



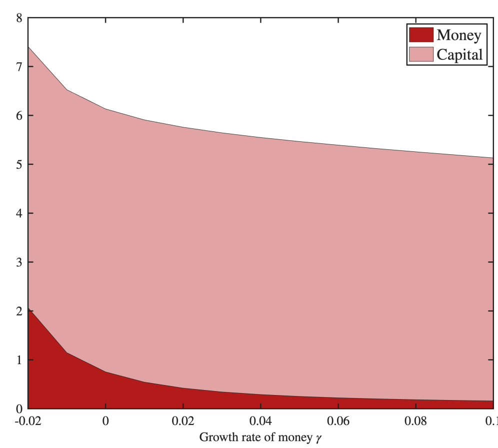
(a) Distribution of offered rates



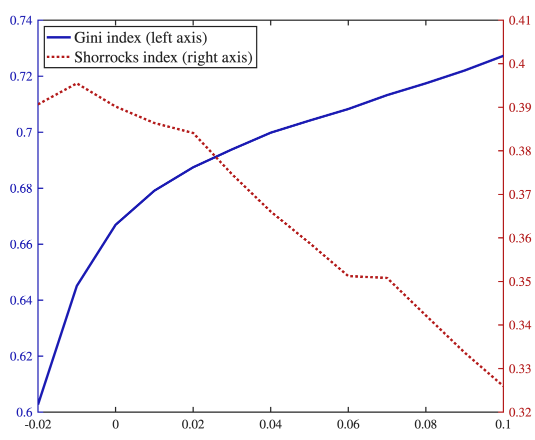
(b) Average earned rates by λ



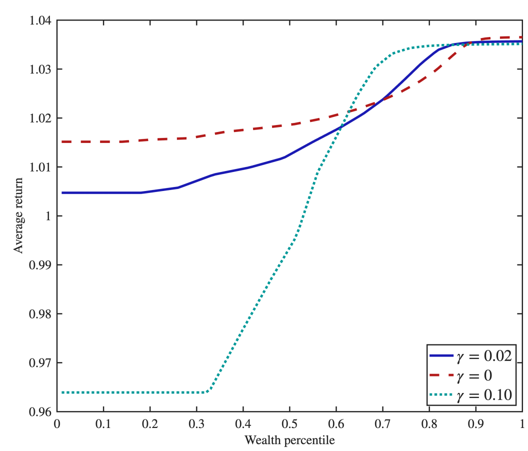
(c) Financial human capital by γ



(d) Wealth, capital and money by γ



(e) Gini coefficient and Shorrocks index by γ



(f) Wealth and returns

Figure 6: Monetary policy

of households across the rungs of the financial human capital ladder. When the growth rate of money is 2%, the fraction of households at λ_1 is 38.3%, the fraction of households at λ_2 is 8.4%, the fraction of households at λ_3 is 14.1%, and the fraction of households at λ_4 is 39.2%. When the growth rate of money is 10%, the fraction of households at λ_1 is 34.0%, the fraction of households at λ_2 is 6.9%, the fraction of households at λ_3 is 11.7%, and the fraction of households at λ_4 is 47.5%.

As the rates of return on money and capital tend to fall, households tend to save less and, for this reason, aggregate wealth declines. Since both aggregate wealth and the fraction of wealth that households hold in cash decline, the real value of the aggregate stock of money falls. Since the decline in aggregate wealth dominates the increase in the fraction of wealth that households invest in capital, the aggregate stock of capital falls. In turn, the fall in the aggregate stock of capital drives the marginal product of capital up. As shown in Proposition 2, the increase in the marginal product of capital leads to an increase in the distribution of interest rates offered by firms—an indirect positive effect that dampens the direct negative effect of inflation on the rates offered by firms. Moreover, the increase in the marginal product of capital leads to smaller increases in the rates offered by firms at the bottom of the distribution and to larger increases in the rates offered by firms at the top of the distribution—an indirect asymmetric effect that compounds the direct asymmetric effect of inflation.

Figure 6(d) shows the long-run effect of the growth rate of money on the aggregate stock of wealth, the real value of the aggregate stock of money, and the aggregate stock of capital. The aggregate stock of wealth declines by 10.9% as the growth rate of money goes from 2 to 10%. The real value of the aggregate stock of money is about 62% lower when the growth rate of money is 10% rather than 2%. The aggregate stock of capital is 6.9% lower when the growth rate of money is 10% rather than 2%.

Figure 6(e) plots the long-run effect of the growth rate of money on wealth inequality, as measured by the Gini coefficient. When the growth rate of money supply increases from 2 to 10%, the Gini coefficient increases from 0.687 to 0.727. The share of wealth owned by the poorest 50% declines from 2.0% to 0.4%. The share of wealth owned by the poorest 75% declines from 17.2% to 13.4%. The share of wealth owned by the poorest 90% declines from 46.1% to 42.7%. Figure 8(a) also plots the long-run effect of the growth rate of money supply on wealth mobility, as measured by the Shorrocks index. The mobility index declines from 0.38 to 0.33. These findings are easy to understand. Inflation lowers the rate of return for households with low financial human capital more than the rate of return for households with high financial human capital. Since households with low financial human capital tend to be poorer than households with high financial human capital, inflation makes the relationship between wealth and rates of return steeper (see Figure 6(f)). For this reason, inflation increases wealth inequality and reduces wealth mobility.

We now turn to the welfare effect of the increase in the growth rate of money supply.

We group households by their state (a, z, λ) at the time of the shock. We compare the average lifetime utility of households with initial state (a, z, λ) before and after the shock. We express the welfare change of households in state (a, z, λ) as the permanent change in consumption that equates their average lifetime utility before and after the shock. We first consider households that are below the 30th percentile of the wealth distribution at the time of the shock. These households have relatively little wealth and, for this reason, they do not suffer much from the decline in the rate of return on money and in the interest rates offered by firms. The shock lowers their welfare by about 2.6%. Second, we consider households that are above the 90th percentile of the wealth distribution at the time of the shock. These households own a lot of wealth and, for this reason, they can be severely affected by the decline in the rate of return on money and in the interest rates offered by firms. These households, however, tend to be at the highest rung of the financial human capital ladder and, for this reason, their rates of return do not fall by much. The shock lowers their welfare by about 4.7%. Third, we consider households that are between the 30th and the 90th percentile of the wealth distribution at the time of the shock. These households typically own some wealth and are at intermediate rungs of the financial human capital ladder. The shock lowers their welfare by about 4.6%. In aggregate, the increase in the growth rate of money supply—and the consequent increase in inflation—lowers welfare by about 4%.

It is worth putting our findings in the context of the literature. Using a model where money is in the utility function and a model where money is a medium of exchange, Lucas (2000) measures the welfare cost of increasing inflation from 0 to 10% to be less than 1% of consumption. The findings in Cooley and Hansen (1989) are similar. These models deliver essentially the same results because, in all of them, the welfare cost of inflation is determined by the money demand curve, an object that is directly observed in the data. Lagos and Wright (2005) consider a model where money is used as a medium of exchange, but, unlike in a cash-in-advance model, buyers and sellers meet bilaterally and bargain over the terms of trade. They show that the welfare cost of increasing inflation from 0 to 10% is about 1% when buyers have all the bargaining power, but rises to 3% when buyers and sellers have the same bargaining power. Intuitively, if the terms of trade are determined by bargaining, inflation exacerbates the buyer's hold-up problem. Indeed, Rocheteau and Wright (2007) show that, if sellers post the terms of trade, the welfare cost of inflation is essentially the same as in Lucas (2020). In our model, the welfare cost of increasing inflation from 2 to 10% is 4%. The cost is high because inflation widens the gap between the marginal product of capital and the return earned by households.

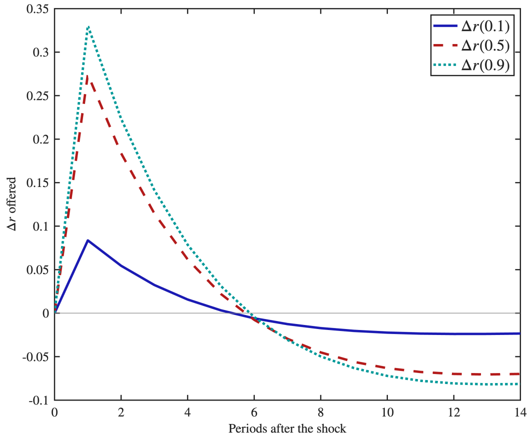
We find that the welfare cost of inflation is similar for poor, middle-class, and rich households. Erosa and Ventura (2002) consider a model with heterogeneous households and find that higher inflation lowers the welfare of poor households only. The difference between our findings and those in Erosa and Ventura (2002) is due to the different role of money. In our model, money is a store of value that households use outside of the capital market. Inflation affects the wedge between the firms' marginal product of capital and households' return on wealth. In Erosa and Ventura (2002), money is a medium of exchange that is used by some households as an alternative to credit, which requires paying a fixed cost. Inflation only hurts poor households, because they are the only ones who find it optimal to use money.

5.2 Productivity shock

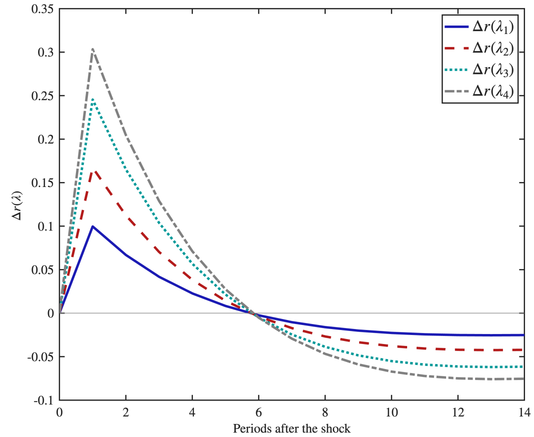
In this subsection, we use the calibrated model to examine the effect of a shock to total factor productivity. Specifically, we consider an unanticipated, transitory increase in total factor productivity A. On impact, total factor productivity A increases by 3%. Over time, total factor productivity follows the autoregressive process $A_{t+1} = \rho A_t + (1-\rho)1$, where $\rho = 0.8$ is the autocorrelation coefficient ($\rho = 0.8$ implies that the half-life of the shock is approximately 3 years). We assume that monetary policy is active, in the sense that it maintains the inflation rate at 2%.

On impact, the increase in total factor productivity increases the marginal product of capital. In turn, the increase in total factor productivity induces firms to offer higher interest rates to households. The increase in the interest rates offered to households is smaller at the bottom of the distribution, where firms are more likely to compete against the households' outside option, and larger at the top of the distribution, where firms are more likely to compete against each other (see Proposition 2). Figure 7(a) plots the time-series of the increase in the interest rate offered by firms at the 10th, the 50th and the 90th percentile of the distribution. On impact, the increase in the interest rate offered by firms at the 10th percentile of the distribution is 8 basis points, the increase at the 50th percentile is 27 basis points, and the increase at the 90th percentile is 33 basis points. At some point during the transition of the economy back to its steady state, the marginal product of capital falls slightly below its stationary value, since households have accumulated additional capital, while the increase in total factor productivity has essentially vanished. At this point, the distribution of interest rates offered by firms slightly declines relative to steady state, and the decline is larger at the top than at the bottom of the distribution.

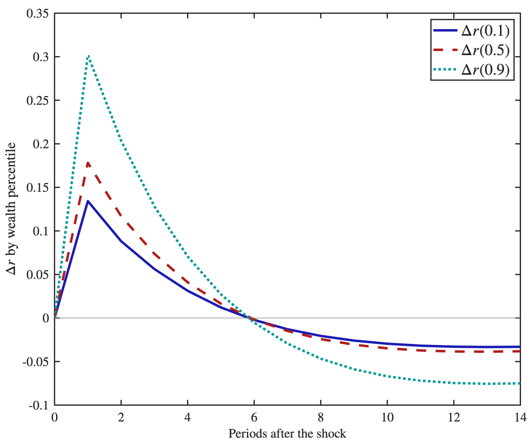
On impact, the stretching out of the distribution of rates offered by firms leads to an asymmetric increase in the rates of return earned by different households. Households at higher rungs of the financial human capital ladder are more likely to lend their savings to firms at the top of the distribution. For this reason, the rate of return earned by these households increases more. Households at lower rungs of the financial human capital ladder are more likely to lend to firms at the bottom of the distribution and they are more likely to hold their savings in cash. For these reasons, the rate of return earned by these households increases less (see Proposition 2). Figure 7(b) plots the time-series of the increase in the rate of return earned by households at the four rungs of the financial human capital ladder. On impact, the return earned by households at the top rung of



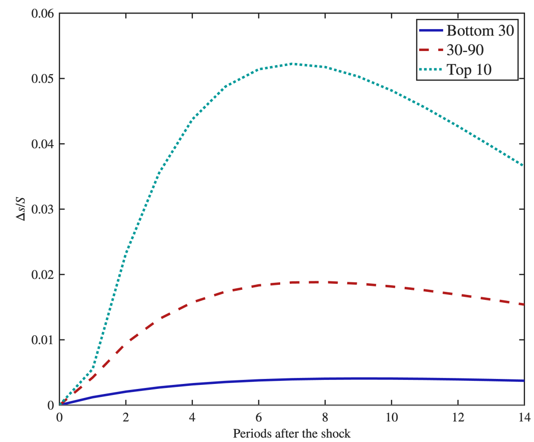
(a) Change in offered rates



(b) Change in earned rates by λ



(c) Change in earned rates by wealth percentile



(d) Change in wealth by initial wealth percentile

Figure 7: TFP Shock

the ladder increases by 30 basis points. The return earned by households at the bottom rung of the ladder increases only by 10 basis points. When, along the transition path, the marginal product of capital falls below its stationary value, the returns earned by households slightly decline relative to steady state, and the decline is larger for households with more financial human capital.

Since wealth and financial human capital are positively correlated, the TFP shock initially leads to a steepening of the relationship between wealth and returns. Figure 7(c) plots the time-series of the increase in the returns earned by households at the 10th, 50th and 90th percentiles of the wealth distribution. On impact, the return earned by households at the 10th percentile of the wealth distribution increases by 13 basis points, the return earned by households at the 50th percentile increases by 18 basis points, the return earned by households at the 90th percentile increases by 30 basis points. The ranking reverses when the marginal product of capital falls below its steady-state value.

Since the of rate of return on their wealth increases, households accumulate more wealth. Since the increase in the rate of return on wealth is larger for households with more wealth, the increase in wealth is larger for richer households. Figure 7(d) plots the additional wealth accumulated by households at different percentiles of the wealth distribution at the time of the shock (i.e., the additional wealth held by households given the shock relative to the wealth they would have held if the shock had not happened). Five years after the shock, the wealth of households at the bottom 30th percentile of the initial wealth distribution increased by 0.4% of average steady-state wealth, the wealth of households between the 30th and the 90th percentile increased by 1.7%, and the wealth of households at the 90th percentile of the initial distribution increased by 4.9%. The positive shock to total factor productivity exacerbates wealth inequality. The fact that richer households benefit more from the shock can be also seen in welfare. On impact, the welfare of households at the bottom 30% of the wealth distribution increases by the equivalent of 0.55% of average consumption, the welfare of households between the 30th and the 90th percentile of the wealth distribution increases by the equivalent of 0.77% of average consumption, and.the welfare of households above the 90th percentile of the wealth distribution increases by 0.82% of average consumption.

5.3 Financial literacy program

Information frictions make the capital market imperfectly competitive and, for this reason, they create distortions in equilibrium outcomes. Since households have limited information about investment opportunities in the capital market, firms can offer interest rates that are below the marginal product of capital. The wedge between the marginal product of capital and the rate of return earned by households creates a negative distortion in aggregate savings. Since households have limited information about investment opportunities, firms can earn positive profits. These profits are not rebated to the households, but they are wasted to finance the entry of an excessively large measure of firms. Overall, imperfect competition in the capital market acts like a "tax" on households' savings, the revenues of which are wasted away.

The extent to which the capital market is imperfectly competitive is not an immutable fact of life, but an endogenous outcome that depends on the households' stock of financial human capital (see Proposition 5). Policies that induce households to accumulate more financial human capital can increase the competitiveness of the capital market and reduce the associated distortions. In this subsection, we consider such a policy. Specifically, we consider a "financial literacy" program that provides, free of charge, a minimal level of investment in financial human capital for all households that are not at the top of the financial human capital ladder. The program is financed through a lump-sum tax on household with total revenues set equal to 1% of output.

Under the literacy program, the long-run distribution of households across the financial human capital ladder becomes essentially degenerate at the top rung. The fraction of households at the bottom rung of the ladder is 0%, the fraction of households at the second rung of the ladder is 0.01%, the fraction of households at the third rung of the ladder is 0.9%, and the fraction of households at the top of the ladder is 99.1%. Without the literacy program, the fractions of households across the four rungs of the financial human capital ladder are 38%, 8%, 14% and 39%.

The literacy program makes competition in the capital market nearly perfect. Intuitively, the financial literacy program allows the vast majority of households to reach the top rung of the financial human capital ladder and, hence, almost all households are in contact with multiple firms. As known from Burdett and Judd (1983), when almost all households are in contact with multiple firms, the market becomes perfectly competitive. Indeed, under the literacy program, the marginal product of capital is 3.4%, the average return earned by households is 2.9%, and the gap between the marginal product of capital and the average return earned by households is 0.5%. In contrast, without the literacy program, the marginal product of capital is 4.3%, the average return earned by households is 1.8%, and the gap between the marginal product of capital and the average return earned by households is 2.5%. Under the literacy program, the fraction of output that is used to finance the entry of firms, which is equal to the quasi-rents earned by firms in the capital market, is 1.3%. Without the literacy program, the fraction of output that is used to finance the entry of firms is 2.2%.

As in a competitive capital market, the relationship between wealth and rates of return becomes flat. On average, households at the 10th, 50th and 90th percentile of the wealth distribution all earn a return of 2.9%. In contrast, without the literacy program, households at the 10th percentile of the wealth distribution earn a rate of return of 0.47%, households at the 50th percentile of the distribution earn a rate of return of 1.25%, and households at the 90th percentile of the distribution earn a rate of return of 3.55%.

The flattening of the relationship between households' wealth and rates of return leads to a reduction in wealth inequality and to an increase in wealth mobility. The poorest

50% of households owns 6.5% of the aggregate wealth, the poorest 90% of households owns 55.4% of the aggregate wealth, and the poorest 99% of households owns 91% of the aggregate wealth. In contrast, without the literacy program, the poorest 50% of households owns 2.0% of the aggregate wealth, the poorest 90% of households owns 46.1% of the aggregate wealth, and the poorest 99% of households owns 88.7% of the aggregate wealth distribution. Thanks to the literacy program, the Gini coefficient of the wealth distribution falls from 0.69 to 0.57, and the Shorrocks mobility index increases from 0.38 to 0.46.

The literacy program effectively eliminates the wedge between the rate of return earned by households and the marginal product of capital and, in so doing, it induces households to accumulate more wealth. Moreover, the literacy program allows almost all households to trade in the capital market and, in doing so, it reroutes wealth from money to capital. Indeed, aggregate wealth increases by 7%, the real value of the aggregate stock of money falls by 71%, and the aggregate stock of capital increases by 13%. In turn, the increase in the aggregate stock of capital leads to a 4.5% increase in aggregate output and to a 2.2% increase in aggregate consumption. The increase in the aggregate stock of capital and in aggregate output give us a measure of the distortions created by imperfect competition in the capital market. The decline in the Gini coefficient and the increase in the Shorrocks index give us a measure of the effect of these distortions on the extent and persistence of wealth inequality.

6 Conclusions

We develop a macroeconomic model in which households face uninsurable risk to their endowment of efficiency units of labor, as in Aiyagari (1994), Bewley (1983), and Huggett (1996), the financial market where households lend their savings and firms borrow capital is subject to information frictions, as in Butters (1977), Varian (1980), and Burdett and Judd (1983), and households invest in their financial human capital (the ability to gather information in the financial market), as in Lusardi, Michaud and Mitchell (2017). The model generates dispersion in interest rates offered by firms (or financial intermediaries) to households for assets with the same risk. The model generates persistent heterogeneity in the returns earned by different households given the same portfolio risk and the same wealth. Wealthier households earn higher returns because they tend to have more financial human capital. We show analytically that the distribution of returns offered by firms, the distribution of rates earned by different households, and the relationship between wealth and returns are endogenous objects and depend on the marginal product of capital, on the inflation rate, and on the distribution of households across the financial human capital ladder. A calibrated version of the model allows us to quantify the effect of monetary, technology, and policy shocks on financial market outcomes, and how changes in financial market outcomes shape the overall response of the economy.

References

[1] Aiyagari, R. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving." *Quarterly Journal of Economics*, 109: 659-684.

[2] Albrecht, J., G. Menzio, and S. Vroman. 2023. "Vertical Differentiation in Frictional Product Markets." *Journal of Political Economy: Macro*, 1: 586-632.

[3] Benhabib, J., A. Bisin, and M. Luo. 2019. "Wealth Distribution and Social Mobility in the US: A Quantitative Approach." *American Economic Review*, 109: 1623-1647.

[4] Benhabib, J., W. Cui, and J. Miao. 2024. "Capital Income Jumps and Wealth Distribution." *Quantitative Economics*, 15: 1197-1247.

[5] Ben-Porath, Y. 1967. "The Production of Human Capital and the Life-Cycle of Earnings." *Journal of Political Economy*, 75: 352-365.

[6] Bewley, T. 1983. "A Difficulty with the Optimum Quantity of Money." *Econometrica*, 51: 1485-1504.

[7] Boar, C., D. Gorea, and V. Midrigan. 2022. "Why Are Returns to Private Business Wealth So Dispersed?" Manuscript, New York University.

[8] Burdett, K., and K. Judd. 1983. "Equilibrium Price Dispersion." *Econometrica*, 51: 955-969.

[9] Burdett, K., and G. Menzio. 2018. "The (Q,S,s) Pricing Rule" *Review of Economic Studies*, 85: 892-928.

[10] Butters, G. 1977. "Equilibrium Distribution of Sales and Advertising Prices." *Review of Economic Studies*, 82: 892-928.

[11] Cagetti, M., and M. De Nardi. 2006. "Entrepreneurship, Frictions, and Wealth." *Journal of Political Economy*, 114: 835–870.

[12] Chatterjee, S. and D. Corbae. 1992. "Endogenous Market Participation and the General Equilibrium Value of Money." *Journal of Political Economy*, 100: 615-646.

[13] Clark, R., A. Lusardi, and O. Mitchell. 2015. "Financial Knowledge and Pension Investment Performance." *Journal of Pension Economics and Finance.*

[14] Cooley, T., and G. Hansen. 1989. "The Inflation Tax and the Business Cycle." *American Economic Review*, 79: 733-748.

[15] Cota, M. and A. Šterc. 2025. "Financial Skills and Search in the Mortgage Market." Manuscript, Nova School of Business and Economics.

[16] Cui, W., R. Wright, and Y. Zhu. 2024. "Endogenous Liquidity and Capital Reallocation." *Journal of Political Economy*, Forthcoming.

[17] Erosa, A., and G. Ventura. 2002. "On Inflation as a Regressive Consumption Tax." *Journal of Monetary Economics*, 49: 761-795.

[18] Fagereng, A., L. Guiso, D. Malacrino, and L. Pistaferri. 2020. "Heterogeneity and Persistence in Returns to Wealth." *Econometrica*, 88: 115-170.

[19] Halvorsen, E., J. Hubmer, S. Ozkan, and S. Salgado. 2024. "Why Are the Wealthiest So Wealthy? New Longitudinal Empirical Evidence and Implications for Theories of Wealth Inequality." Manuscript, University of Pennsylvania.

[20] Head, A., L. Liu, G. Menzio, and R. Wright. 2012. "Sticky Prices: A New Monetarist Approach." *Journal of the European Economic Association*, 10: 939-973.

[21] Hortaçsu, A., and C. Syverson. 2004. "Product Differentiation, Search Costs, and Competition in the Mutual Fund Industry: A Case Study of S&P 500 Index Funds." *Quarterly Journal of Economics*, 119: 403-456.

[22] Hubmer, J., and L. Nord. 2025. "Investment in Demand and Dynamic Competition for Customers." Manuscript, University of Pennsylvania.

[23] Huggett, M. 1996. "The Wealth Distribution in Life-Cycle Economies." *Journal of Monetary Economics*, 38: 469-494.

[24] Jappelli, T., and M. Padula. 2013. "Investment in Financial Knowledge and Saving Decisions." *Journal of Banking and Finance*, 37: 2779-2792.

[25] Kaplan, G., and G. Menzio. 2016. "Shopping Externalities and Self-Fulfilling Unemployment Fluctuations." *Journal of Political Economy*, 124: 771-825.

[26] Kaplan, G., and G. Violante. 2014. "A Model of the Consumption Response to Fiscal Stimulus Payments." *Econometrica*, 82: 1199-1239.

[27] Kaplan, G., B. Moll, and G. Violante. 2018. "Monetary Policy According to HANK." *American Economic Review*, 108: 697-743.

[28] Lagos, R., and R. Wright. 2005. "A Unified Framework for Monetary Theory and Policy Analysis." *Journal of Political Economy*, 113: 463- 484.

[29] Lagos, R., and S. Zhang. 2020. "Turnover Liquidity and the Transmission of Monetary Policy." *American Economic Review*, 110: 1653-1672.

[30] Lagos, R., and S. Zhang. 2022. "The Limits of ONETARY ECONOMICS: On Money as a Constraint on Market Power." *Econometrica*, 90: 1177-1204.

[31] Lucas, R. 2000. "Inflation and Welfare." *Econometrica*, 68: 247-274.

[32] Lusardi, A., P. Michaud, and O. Mitchell. 2017. "Optimal Financial Knowledge and Wealth Inequality." *Journal of Political Economy*, 125: 431-477.

[33] Lusardi, A., and O. Mitchell. 2023. "The Importance of Financial Literacy: Opening a New Field." *Journal of Economic Perspectives*, 37: 137-154.

[34] McKay, A. 2013. "Search for Financial Returns and Social Security Privatization." *Review of Economic Dynamics*, 16: 253-270.

[35] Menzio, G. 2023. "Optimal Product Design: Implications for Competition and Growth under Declining Search Frictions." *Econometrica*, 91: 605-639.

[36] Menzio, G. 2024. "Markups: A Search-Theoretic Perspective." NBER Working Paper 32888.

[37] Menzio, G., and S. Spinella. 2025. "More on onetary Economics," Manuscript, New York University.

[38] Nord, L. 2023. "Shopping, Demand Composition, and Equilibrium Prices." Manuscript, University of Pennsylvania.

[39] Pytka, K. 2018. "Shopping Effort in Self-Insurance Economies." Manuscript, Mannheim University.

[40] Rocheteau, G., and R. Wright. 2005. "Money in Search Equilibrium, Competitive Equilibrium, and Competitive Search Equilibrium." *Econometrica*, 73: 175-202.

[41] Sangani, K. 2023. "Markups Across the Income Distribution: Measurement and Implications." Manuscript, Harvard University.

[42] Silveira, R., and R. Wright. 2016. "Venture Capital: A Model of Search and Bargaining." *Review of Economic Dynamics*, 19: 232-246.

[43] Varian, H. 1980. "A Model of Sales." *American Economic Review*, 70: 651-659.

[44] Woodward, S., and R. Hall. 2012. "Diagnosing Consumer Confusion and Sub-optimal Shopping Effort: Theory and Mortgage-Market Evidence." *American Economic Review*, 102: 3249-3276.